

X-ray and neutron sources

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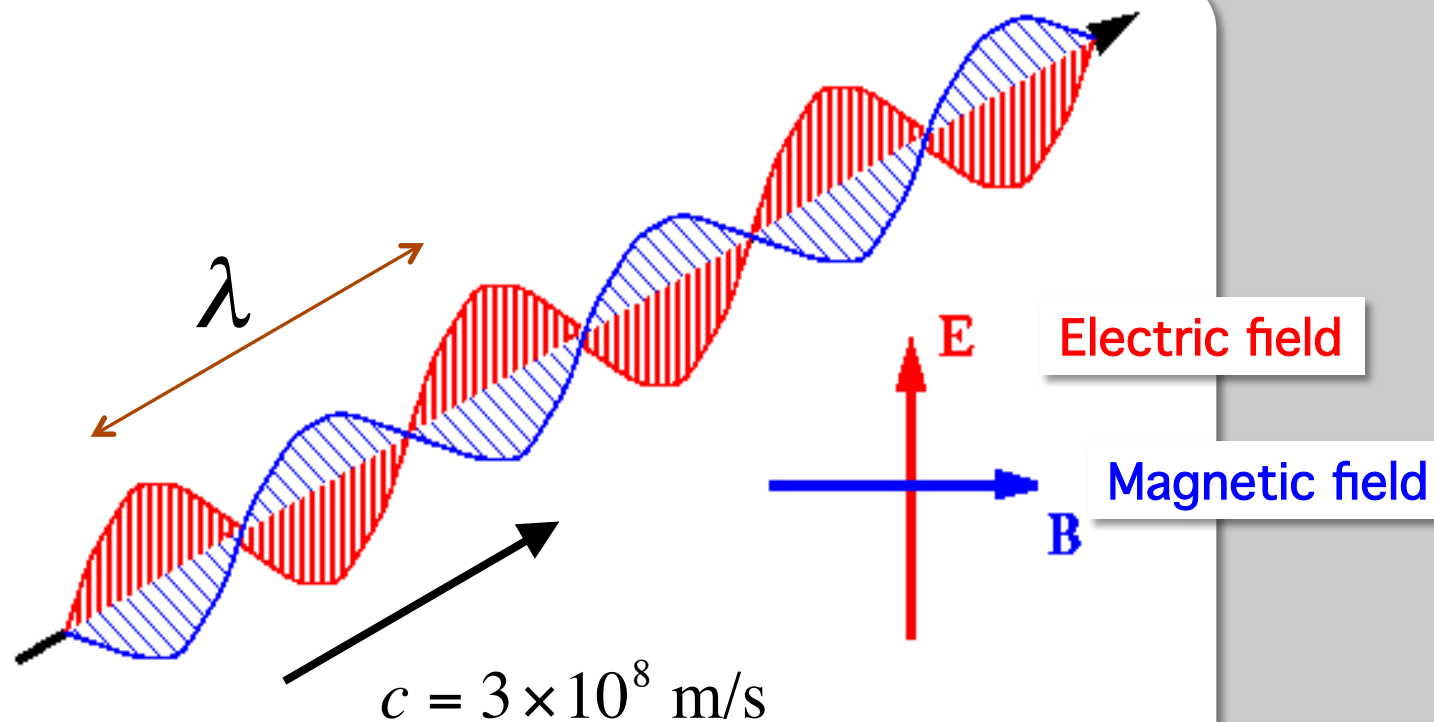
Insubria International Summer School on
Crystallography for Health and Biosciences
Como (Italy), June, 18th – 23rd 2012

Overview

1. Mechanisms of X-ray production
2. Laboratory X-ray sources
3. Synchrotron Radiation
 - SR sources
 - SR from bending magnets
 - SR from insertion devices
4. Neutron sources

Mechanisms of X-ray production

X-rays are electromagnetic radiation



Superposition
of sinusoidal waves

X-rays

$$\lambda \approx 0.01 \div 10 \text{ \AA}$$

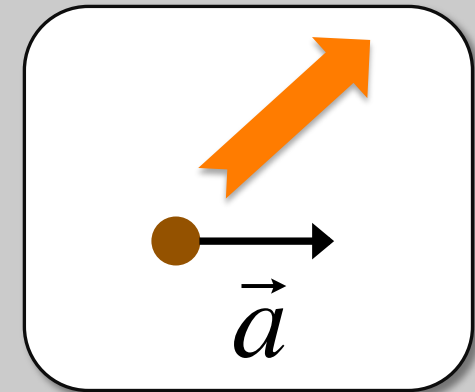
$$\nu \approx 10^{17} \div 10^{20} \text{ Hz}$$

$$E \approx 1 \div 400 \text{ keV}$$

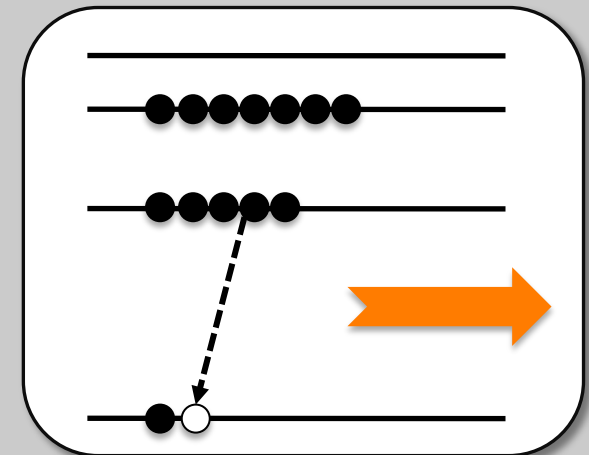
Two mechanisms ...

... of production of electromagnetic radiation

1. Emission from accelerated electric charges

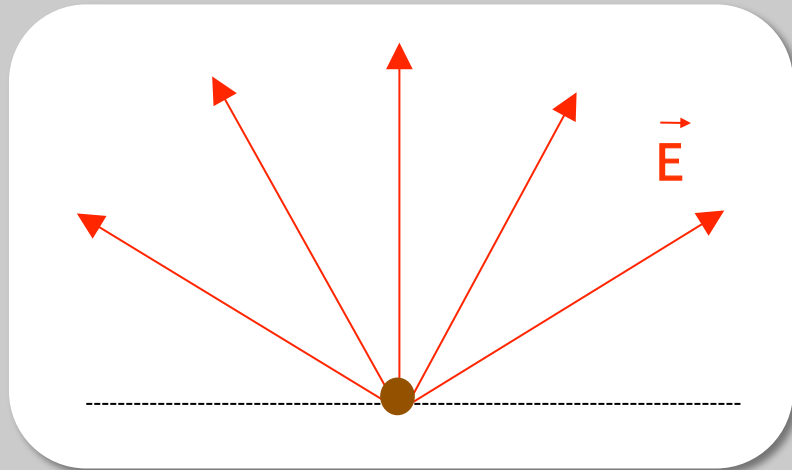


2. Emission as effect of quantum transitions

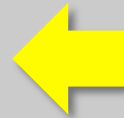


Non-accelerated charges

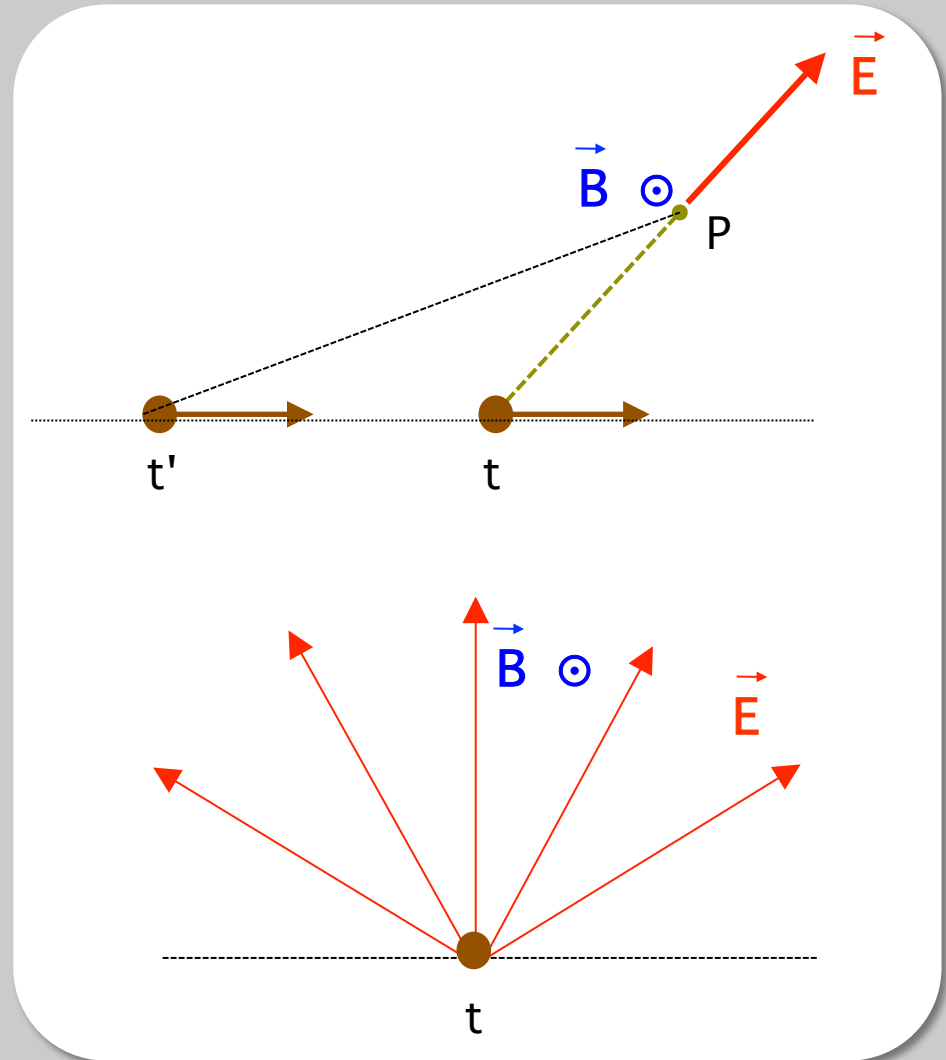
Charge at rest



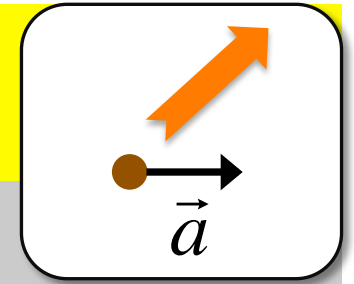
No power is emitted
as electromagnetic radiation



Constant velocity



Accelerated charge: a tutorial example (a)



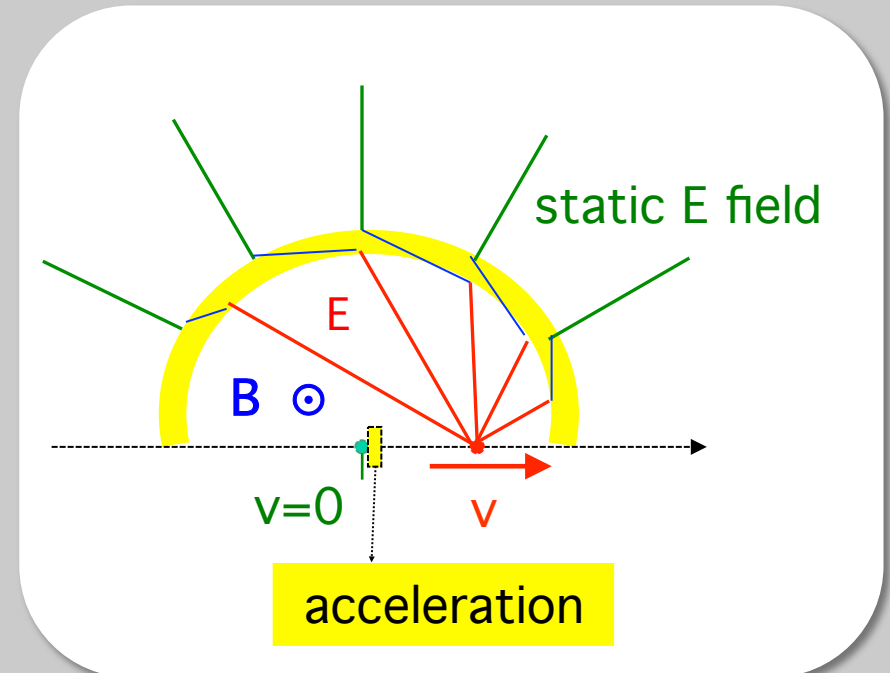
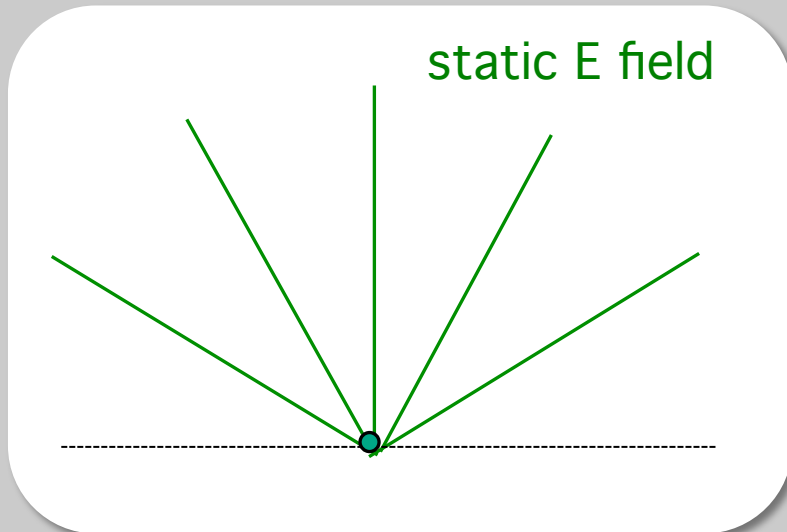
Charge at rest



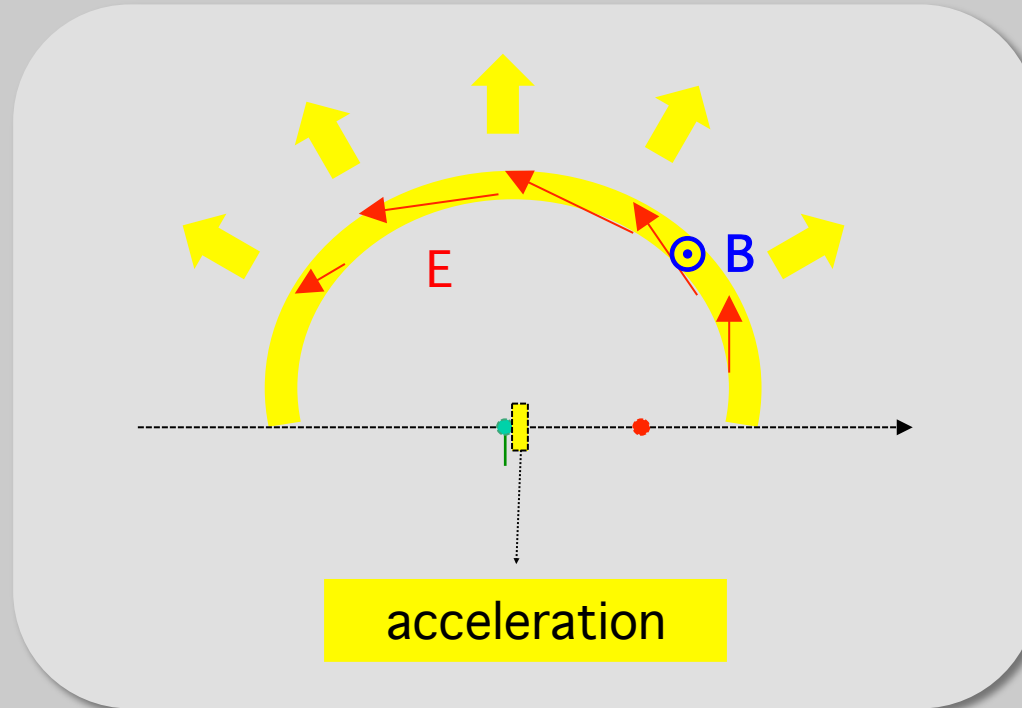
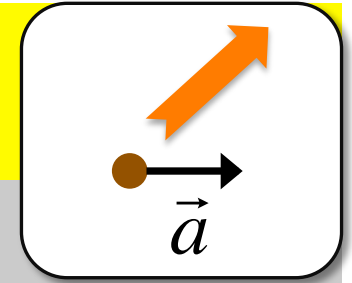
Short acceleration



Constant velocity



Accelerated charge: a tutorial example (b)



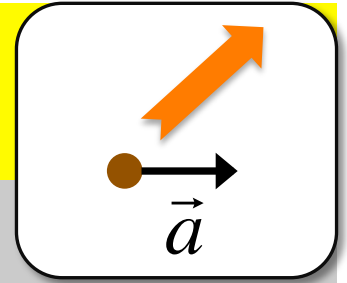
Acceleration



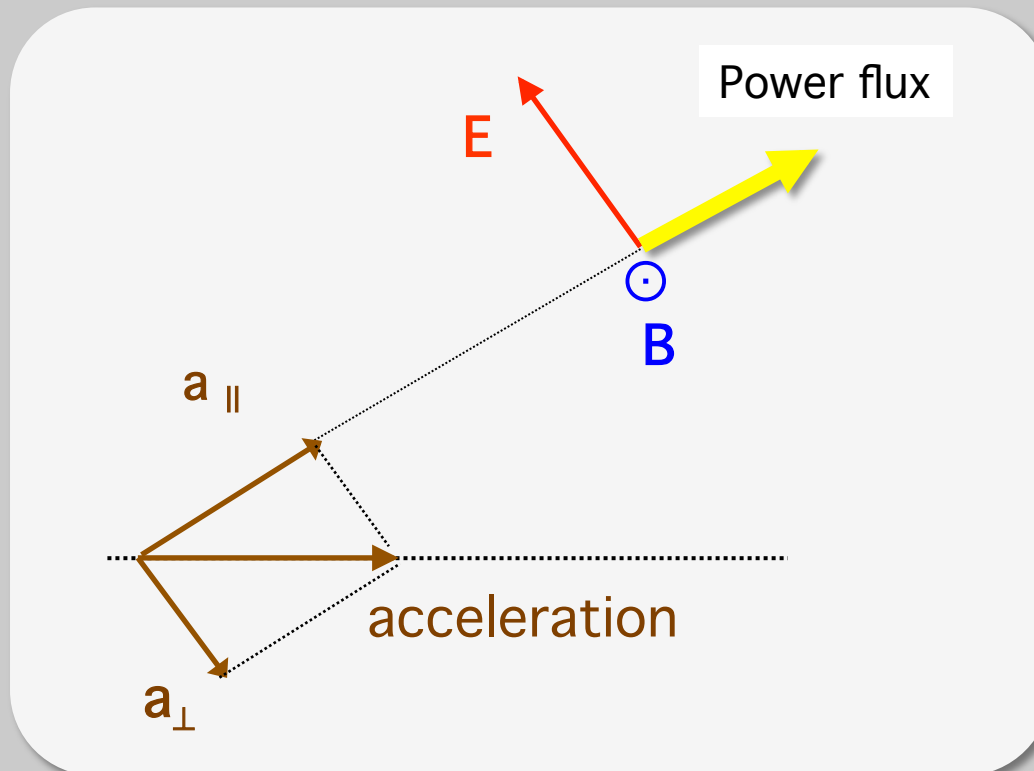
Perturbation
of static fields

- Electro-magnetic field
- Propagating with velocity $c = 3 \times 10^8$ m/s
- Carrying energy

Dipole approximation



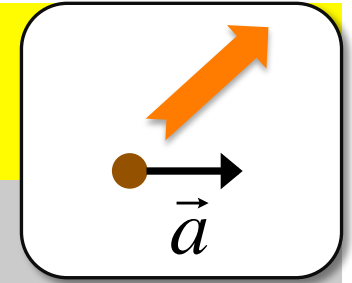
- charge velocity: $v \ll c$
- charge size: $d \ll \lambda$
- observer distance: $r \gg \lambda$



$$\vec{E}(\vec{r}, t) = -\frac{q\vec{a}_{\perp}(t')}{4\pi\epsilon_0 r c^2}$$

$$\vec{B}(\vec{r}, t) = \frac{\hat{r} \times \vec{E}(\vec{r}, t)}{c}$$

Dipole approximation: radiated power

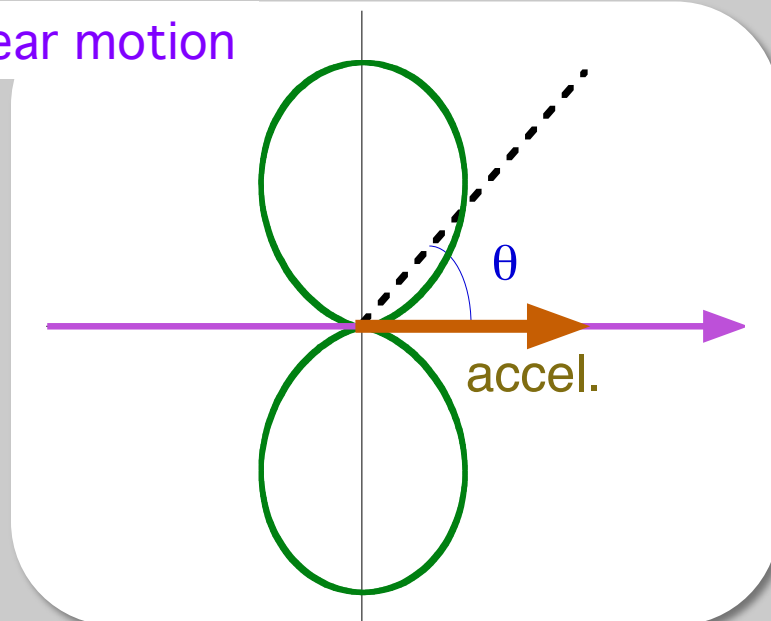


The emitted power is:

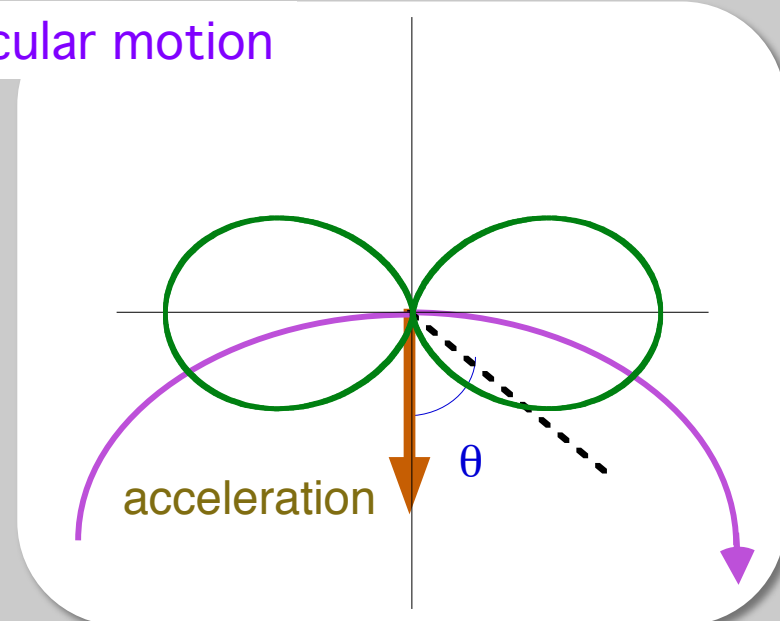
- Zero in the direction of acceleration
- Maximum in the perpendicular plane

Angular emission profiles

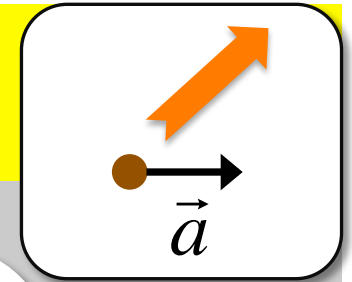
Linear motion



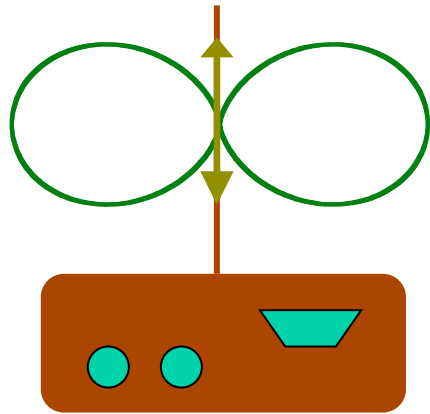
Circular motion



Examples

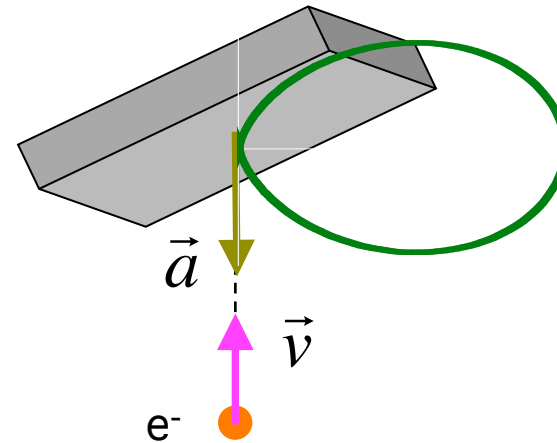


Oscillating motion



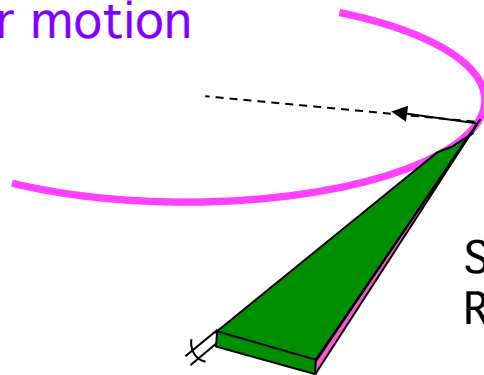
Broadcasting antenna

Average linear deceleration



Bremsstrahlung
in X-ray tubes

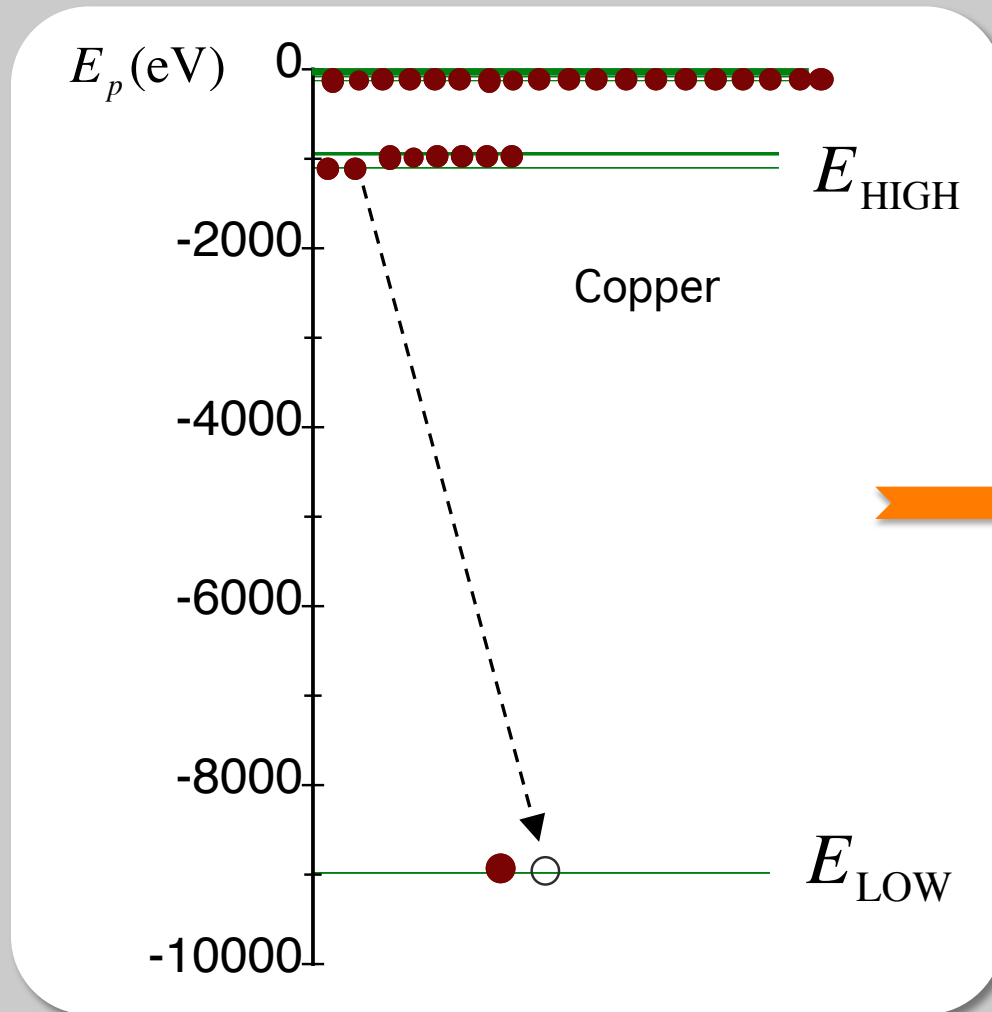
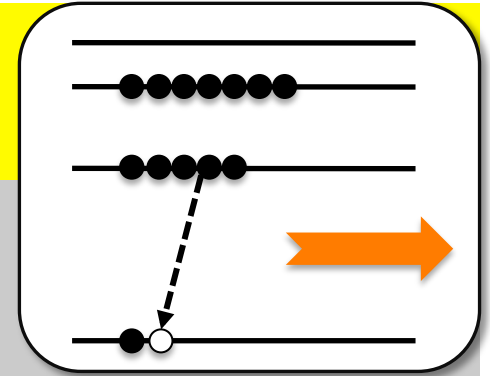
Relativistic circular motion



Synchrotron
Radiation

No dipole approximation
Relativistic treatment necessary

Quantum transitions

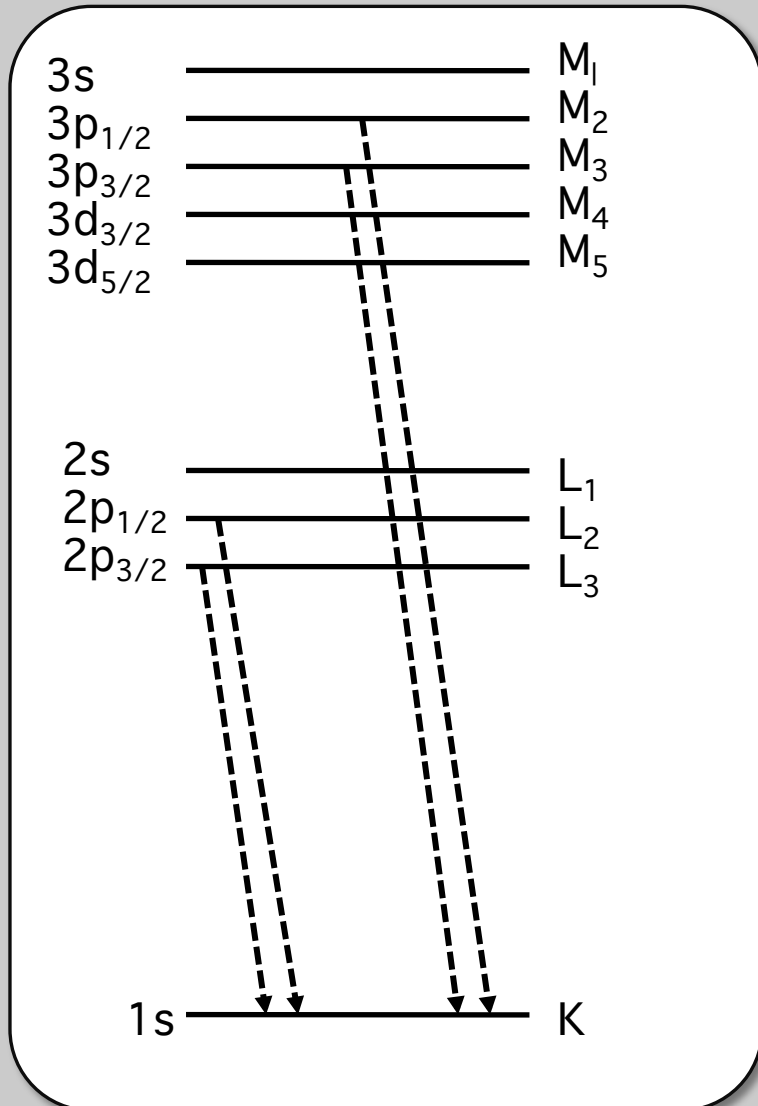
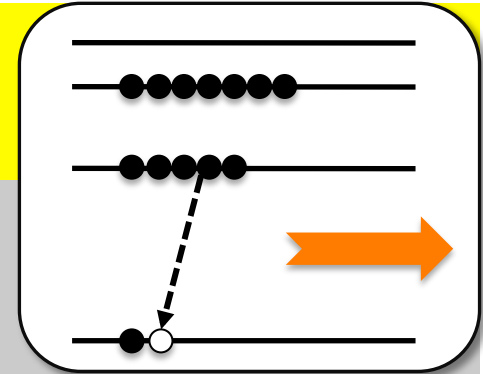


$$\hbar\omega = E_{\text{HIGH}} - E_{\text{LOW}}$$

Atomic X-ray transitions

$$\hbar\omega \approx 0.4 \div 100 \text{ keV}$$

Characteristic lines



Transitions and X-ray lines

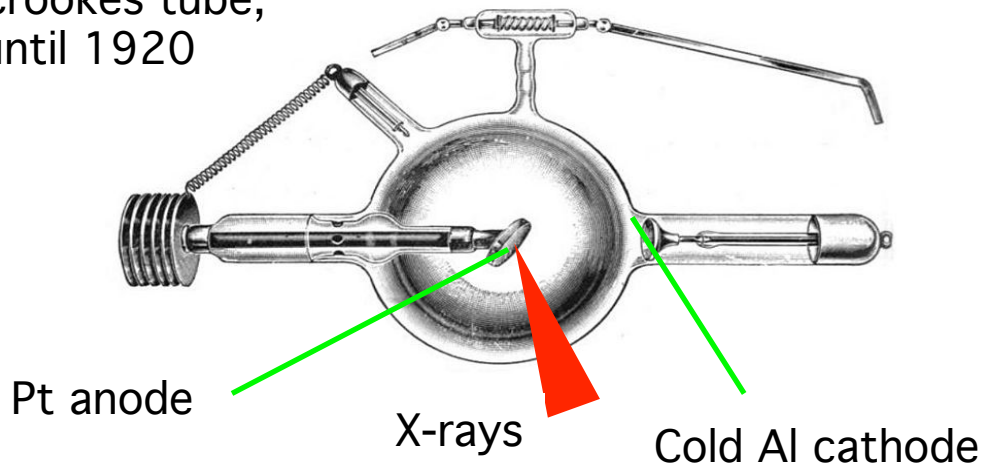
K - L ₃	Kα ₁	Kα
K - L ₂	Kα ₂	
K - M ₃	Kβ ₁	Kβ
K - M ₂	Kβ ₃	

(Electric-dipole allowed transitions)

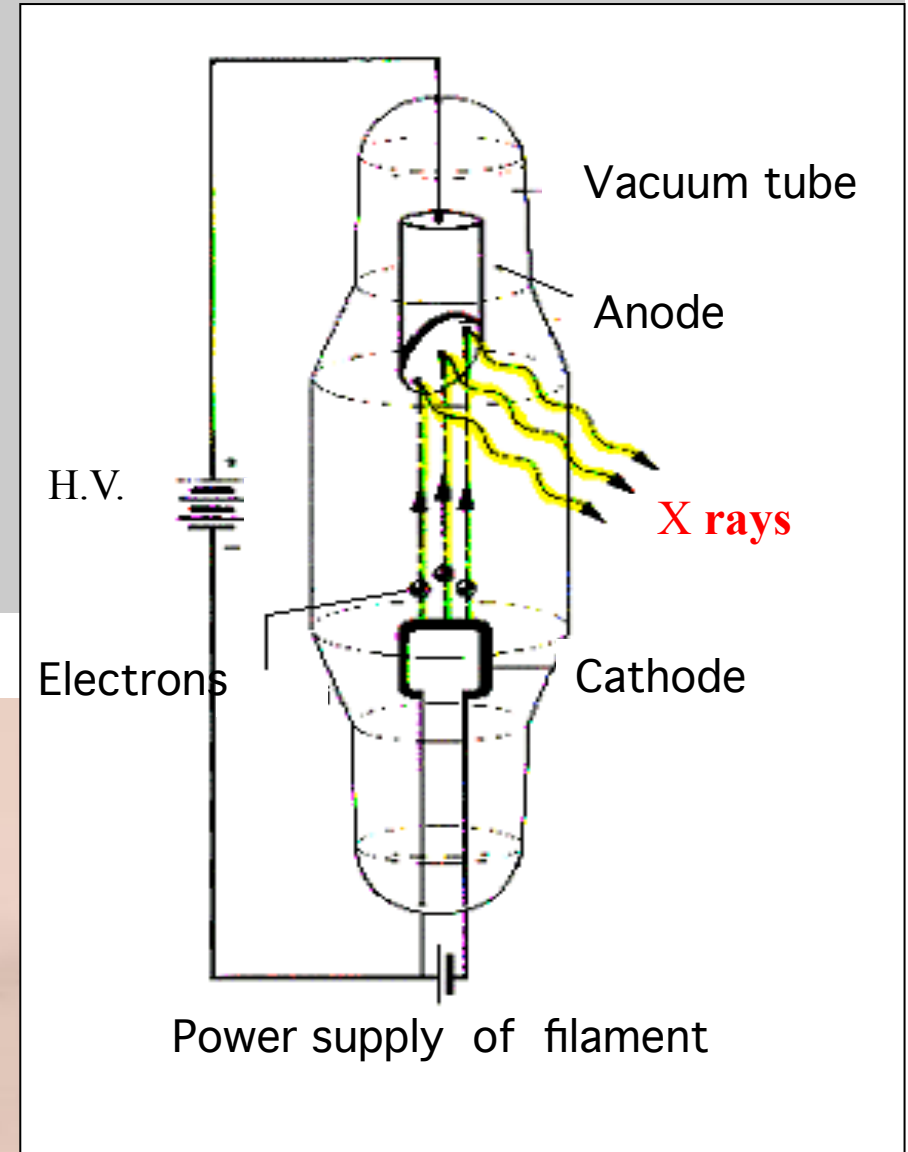
Laboratory X-ray sources

X-ray tubes

Crookes tube,
until 1920



Hot cathode Coolidge's tube (1913)



Present-day laboratory tubes



High vacuum sealed tubes

Independent control of I and V

Accelerating voltage < 100 kV

Different anode metals:
Cr, Cu, Mo, W, ...

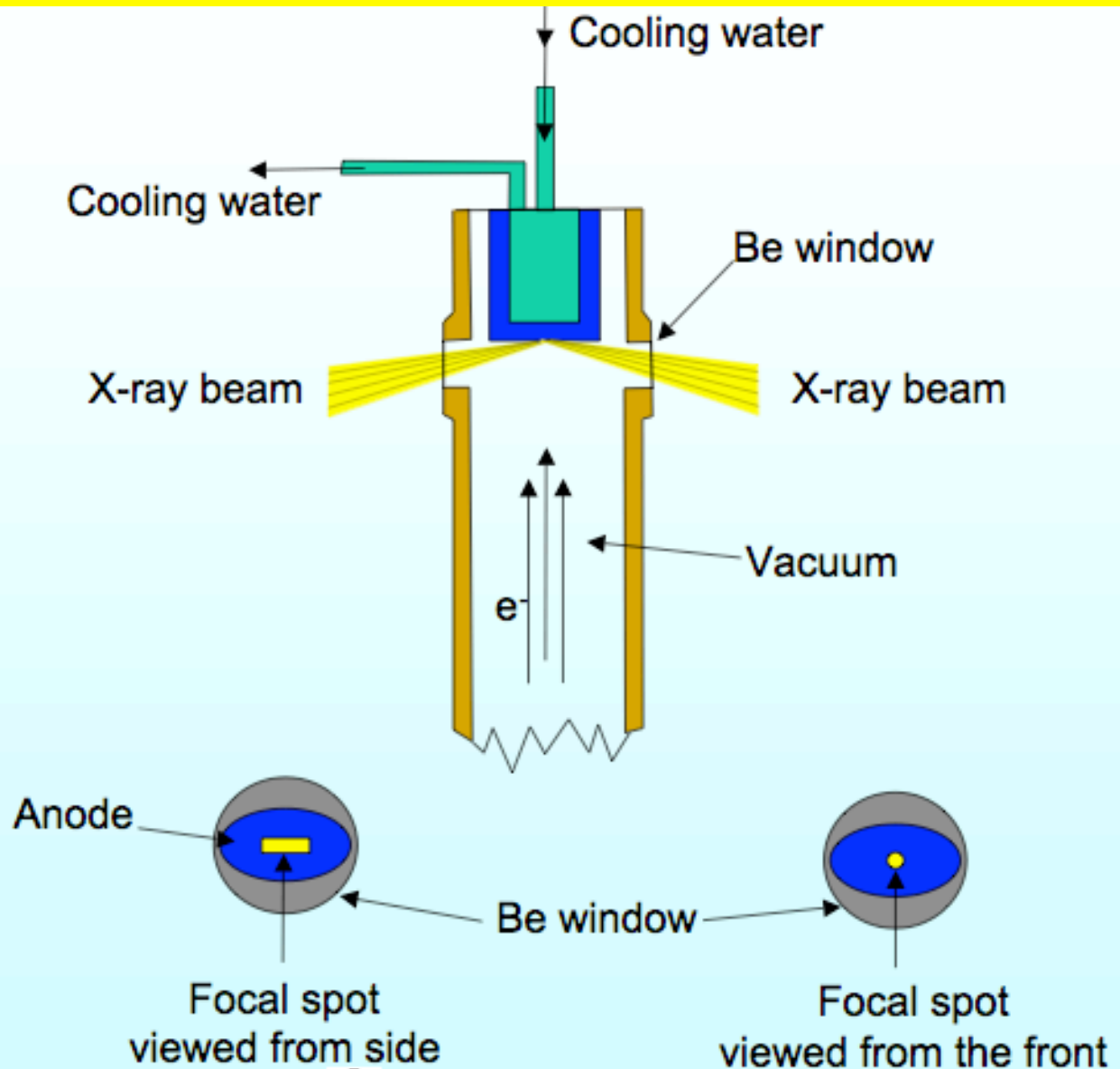
Power: 1 - 4 kW

~ 1 % X-ray production

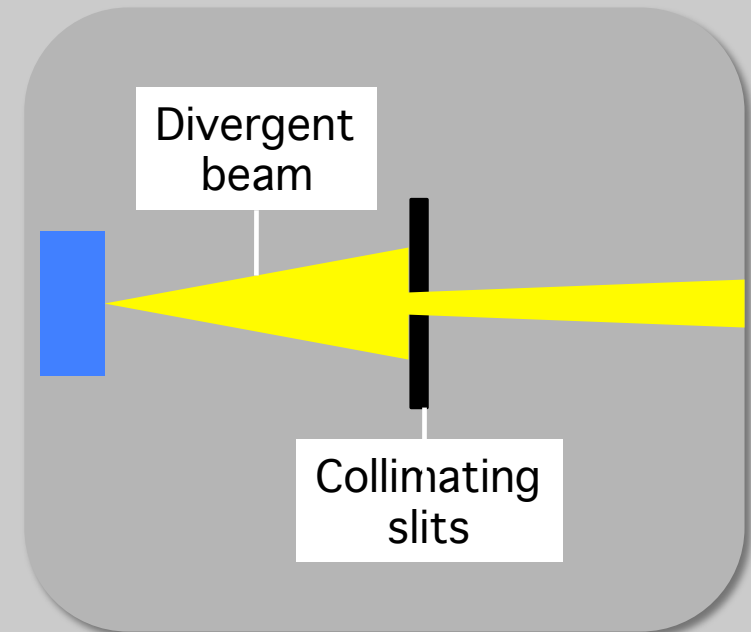
~99 % anode heating

→ water cooling

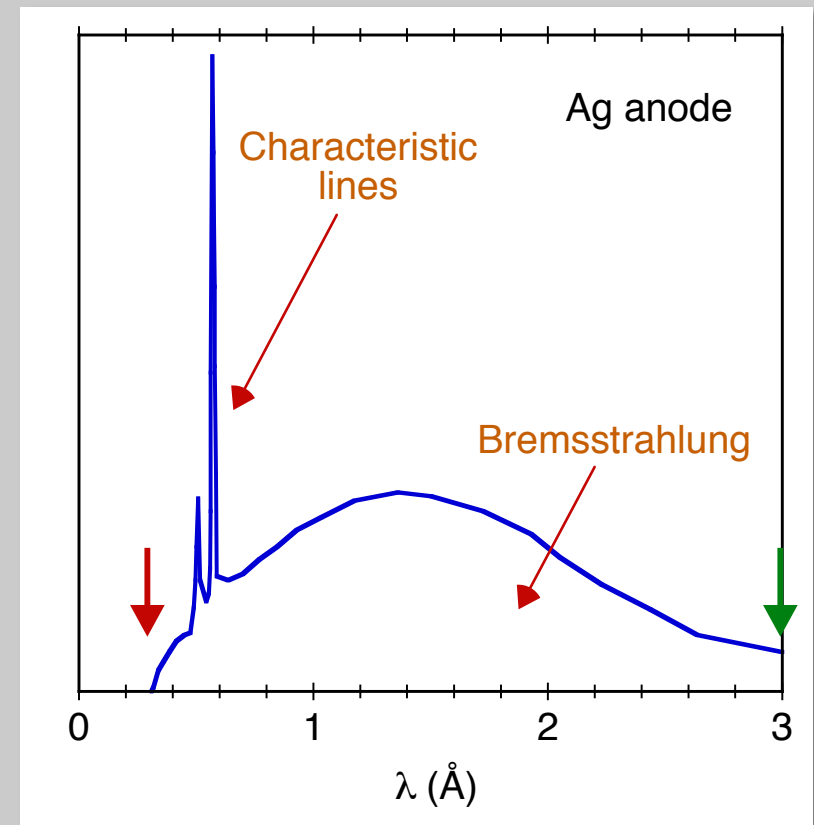
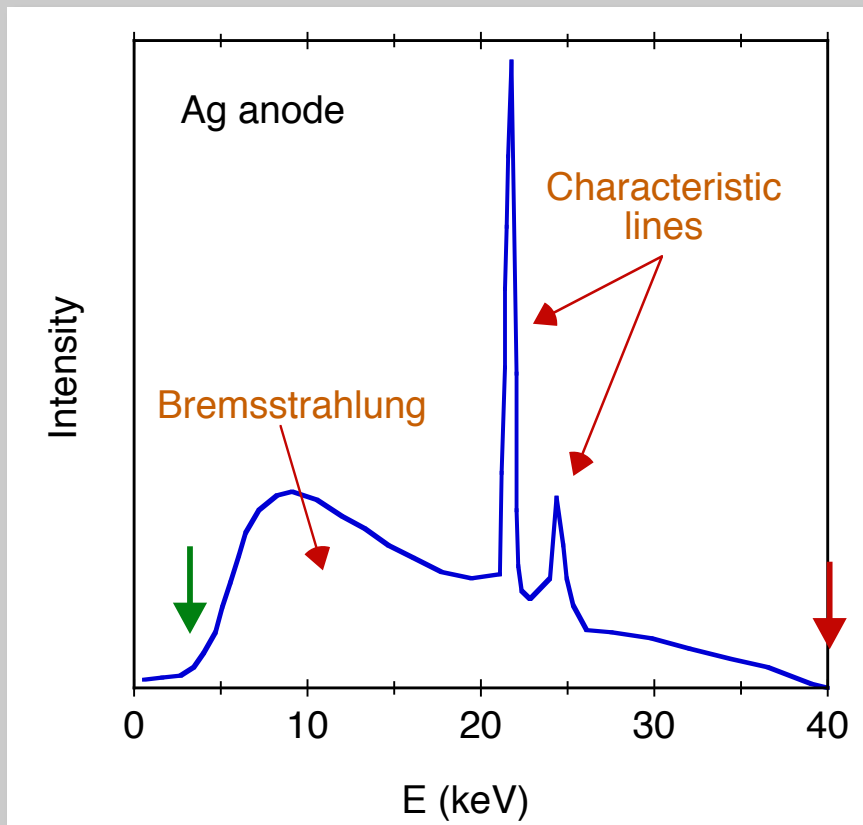
X-ray emission



Un-polarized X-rays

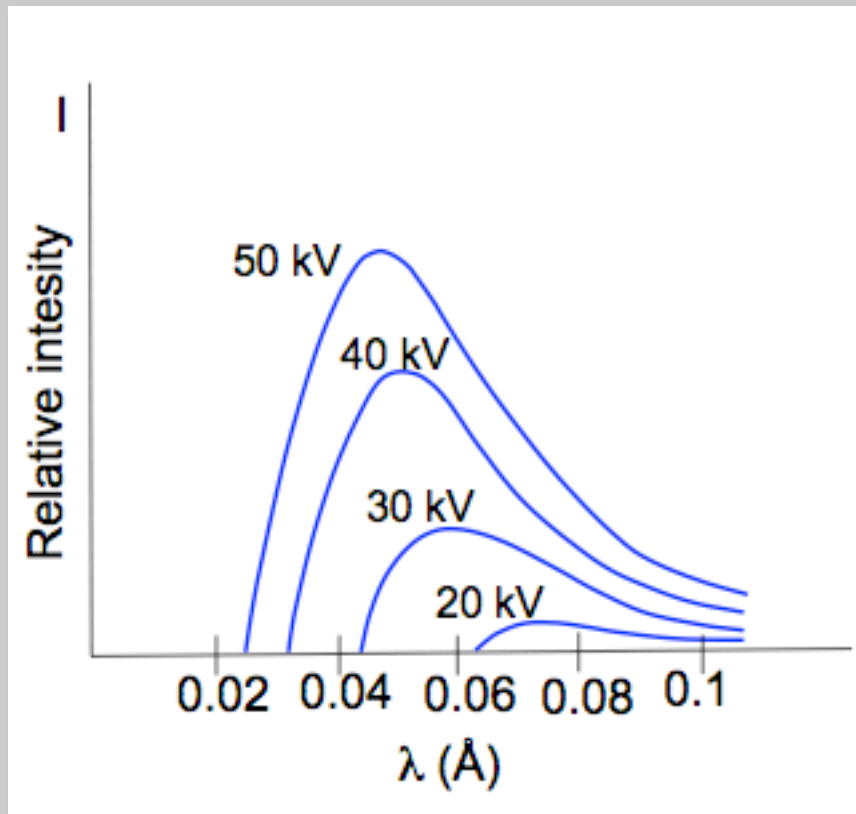


Emission spectrum



$$\lambda [\text{Å}] = \frac{12.4}{E [\text{keV}]}$$

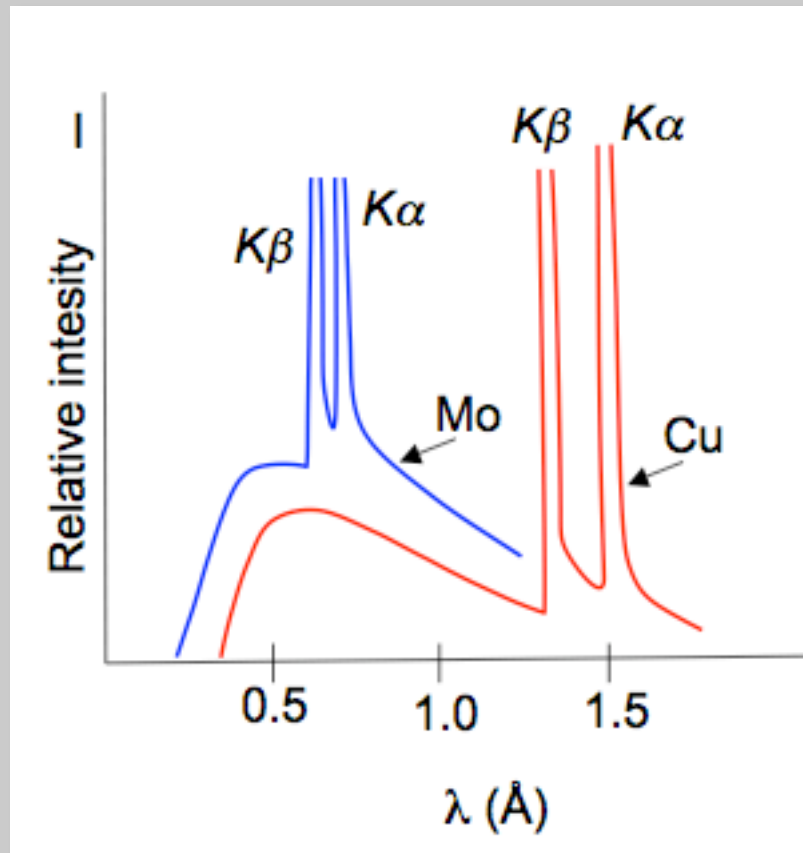
Continuous bremsstrahlung spectrum



Electrons accelerating voltage:

- Maximum energy (minimum λ)
- Total intensity

Characteristic lines



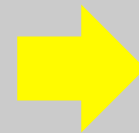
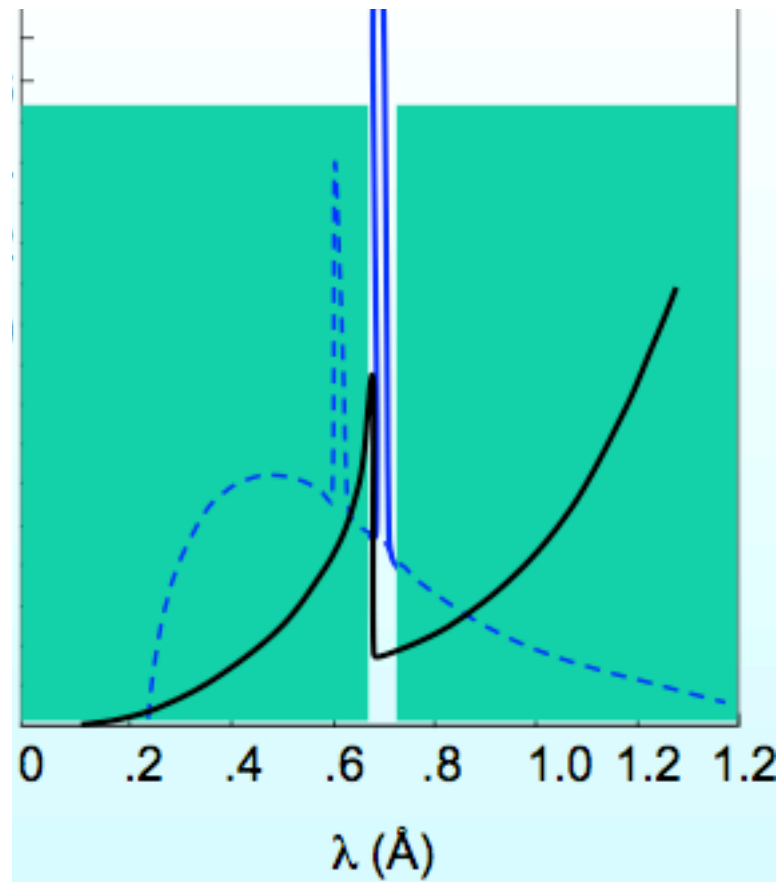
Moseley law

$$\lambda \propto \frac{1}{Z^2}$$

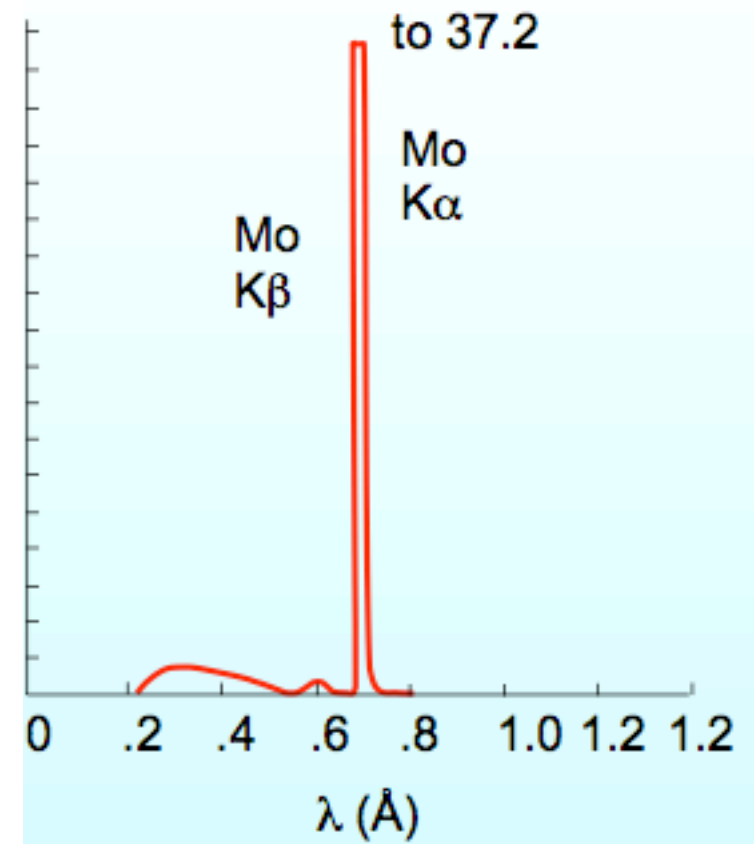
		Cr	Cu	Mo
$K\alpha$	$K\alpha_1$	2.2897	1.5406	0.7093
	$K\alpha_2$	2.2936	1.5444	0.7136
$K\beta$		2.0849	1.3922	0.6323

Filters

- Mo emission spectrum
- Zr absorption coefficient



- Mo filtered spectrum

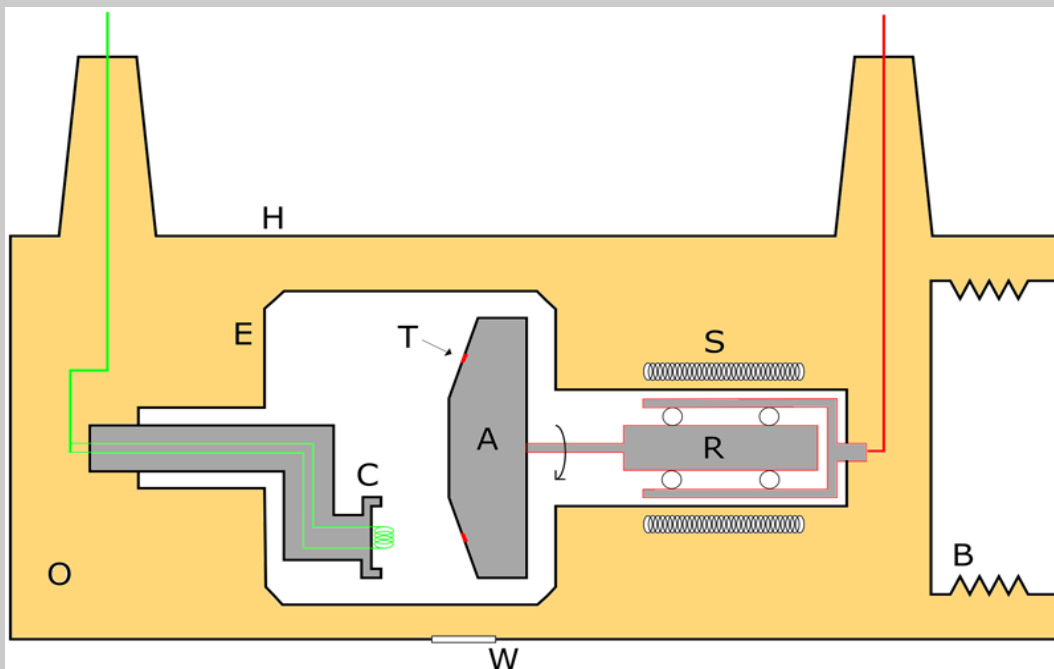


Rotating-anode tubes



Snapshot emission:
no liquid cooling

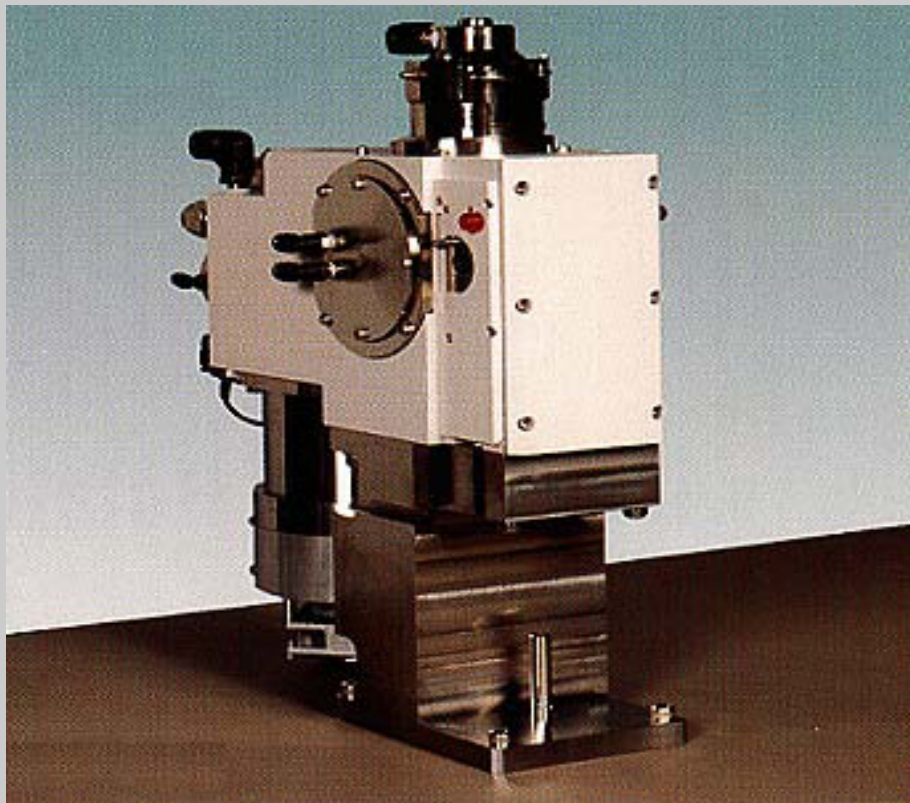
Power ratings up to 100 kW



Anode rotated by
electromagnetic induction

Almost all medical tubes
are rotating anode tubes
(exception: dental tubes)

Liquid cooled rotating-anode sources



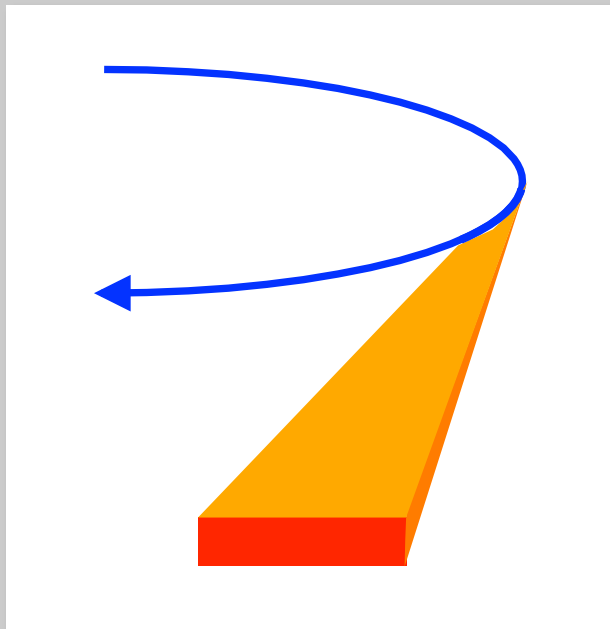
Power ratings 12 – 18 kW

For non-snapshot applications: diffraction, tomography, etc.

Synchrotron Radiation

Synchrotron radiation

Electromagnetic radiation emitted by centripetally accelerated electrons moving at relativistic speed



Storage rings

Relative velocity

$$\beta = \frac{v}{c} \approx 1$$

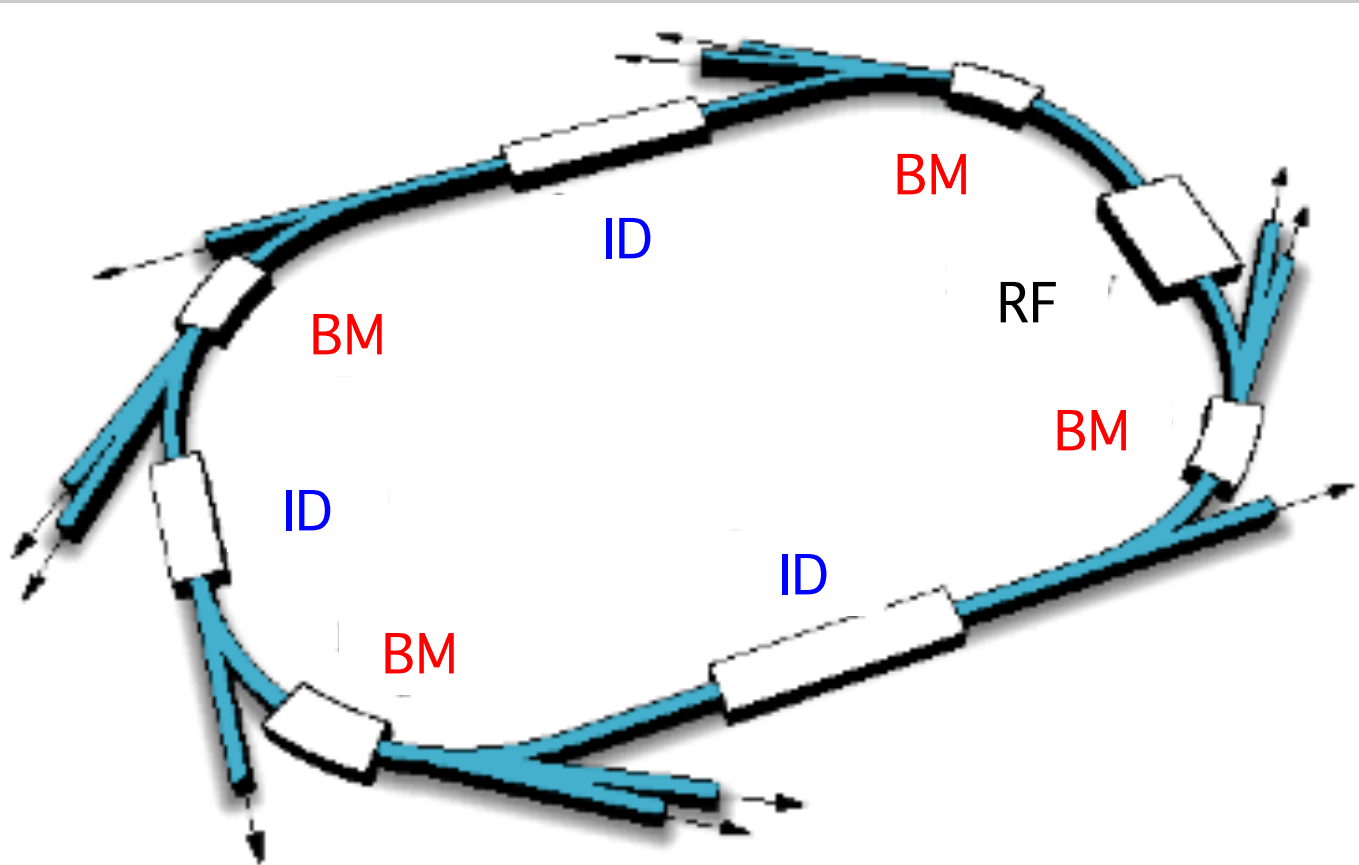
Electron energy

$$W \cong 1 \div 10 \text{ GeV}$$

A key parameter

$$\begin{aligned} \gamma &= \left(1 - v^2 / c^2\right)^{-1/2} \\ &= W / m_0 c^2 \\ &\approx 2000 \div 20000 \end{aligned}$$

Storage rings as S.R. sources



Basic components

BM = bending magnets

ID = insertion devices

RF = radiofrequency cavity

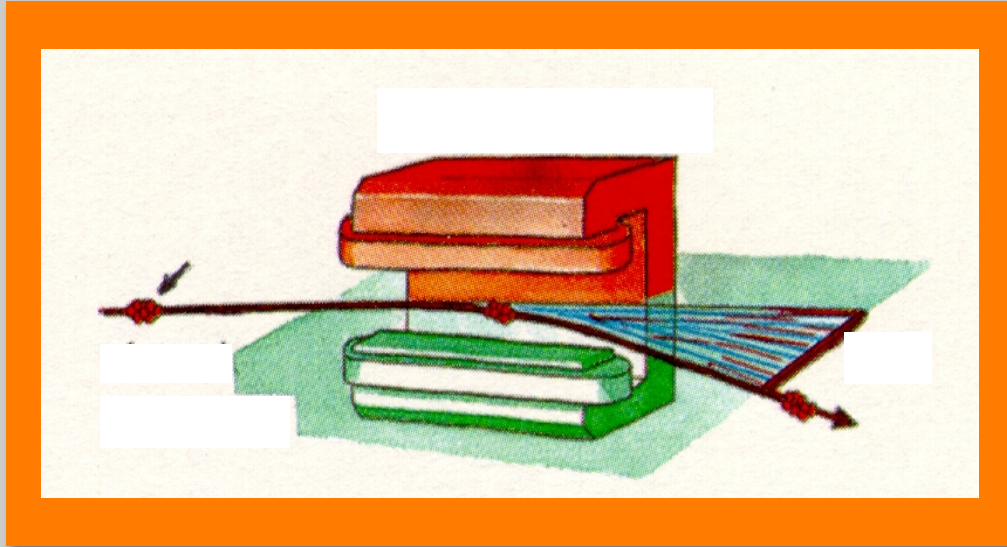
Synchrotron Radiation from:

➤ **Bending magnets**

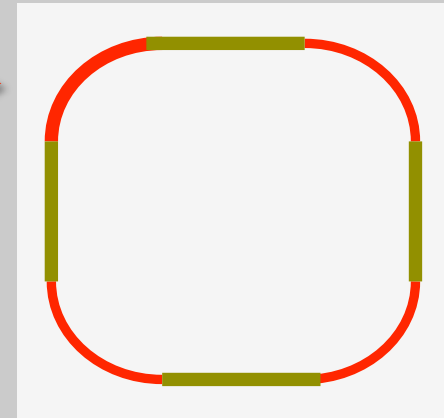
➤ **Insertion devices ...**

- wigglers
- undulators

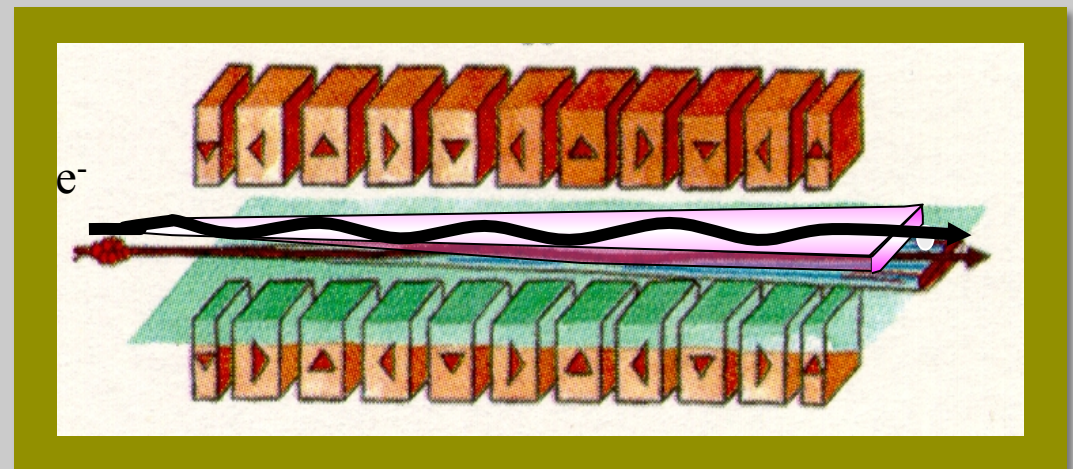
Bending magnets and insertion devices



Bending
magnet



Insertion
device



S.R. Facilities - map



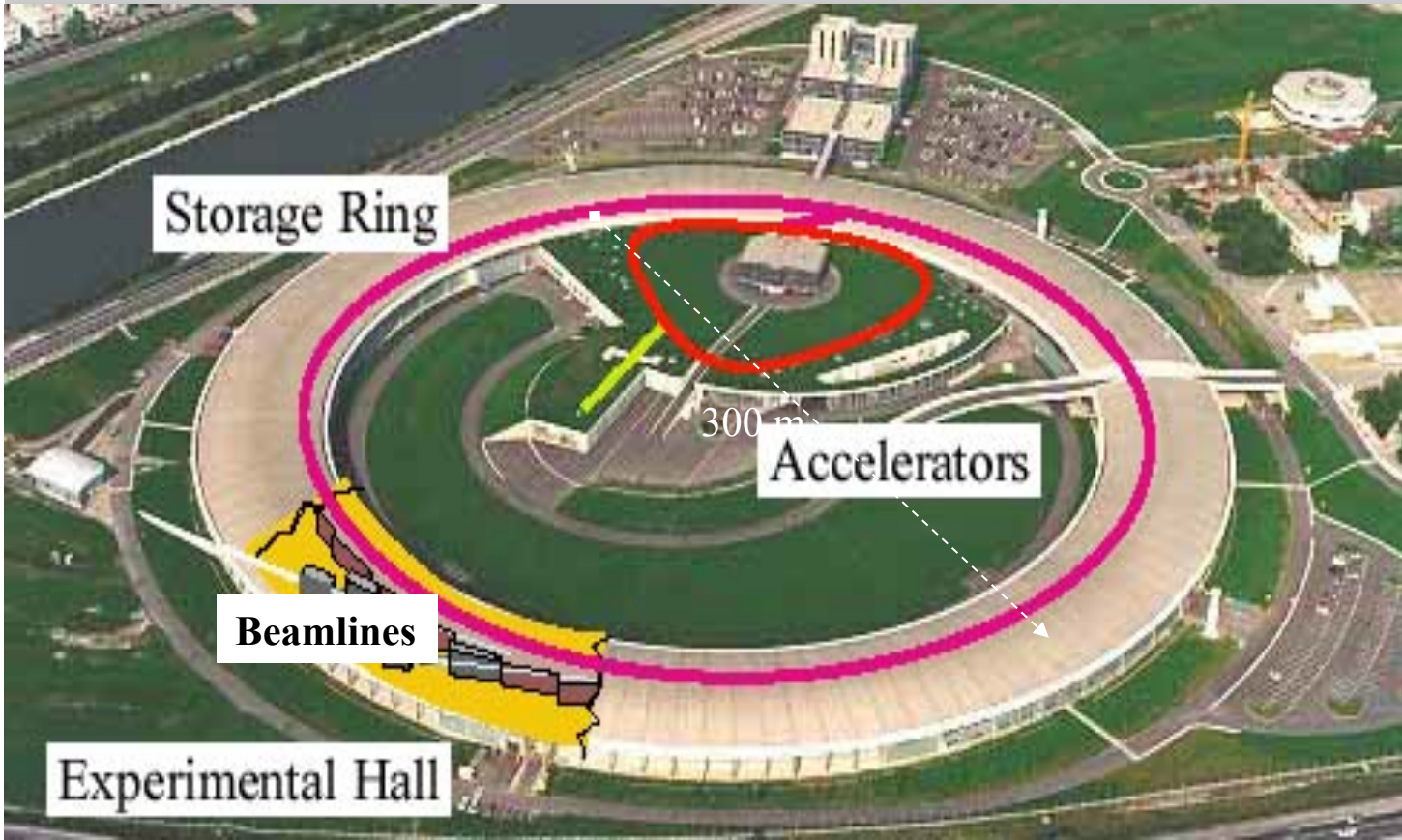
S.R. Facilities

Name	Site	Year	E (GeV)
SPring8 Super Photon ring 8 GeV	Hyogo (Japan)	1997	8
APS Advanced Photon Source	Argonne, IL (USA)	1996	7
ESRF European S. R. Facility	Grenoble (France)	1994	6



Name	Site	Year	E (GeV)
PETRA III	Hamburg (D)	2009	6
DORIS III	Hamburg (D)	1980	4.45
Diamond	Didcot (UK)	2007	3
Soleil	S.Aubin (F)	2006	2.75
Elettra	Trieste (I)	1994	2.4
ALBA	Barcelona (E)	2012	3.0
SLS	Villigen (CH)	2001	2.4

E.S.R.F = European Synchrotron Radiation Facility



Electron energy

$$W = 6 \text{ GeV}$$

$$\gamma = 12000$$

Diameter

300 m

32 bending magnets
32 straight sections

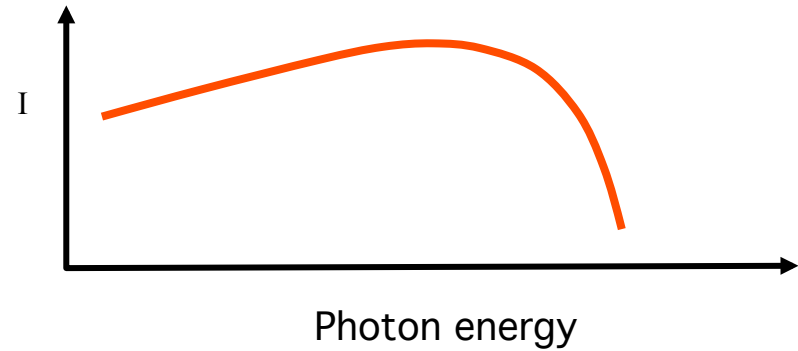
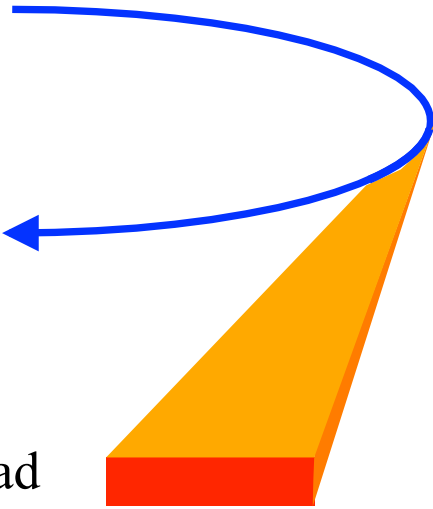
15 BM beamlines
32 ID beamlines

S. R. from bending magnets

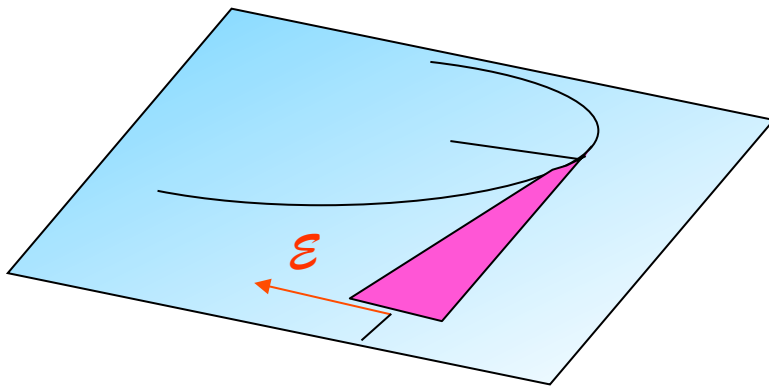
Properties of Synchrotron Radiation

Collimation

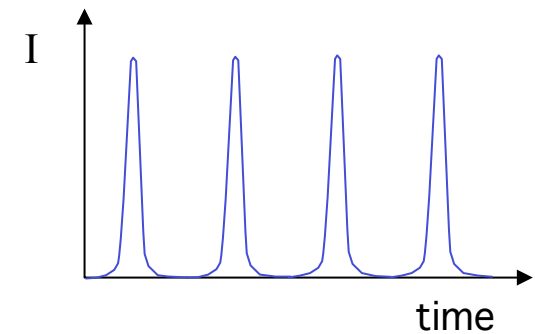
$$\Theta \cong 1/\gamma \text{ rad}$$



High intensity
continuous spectrum



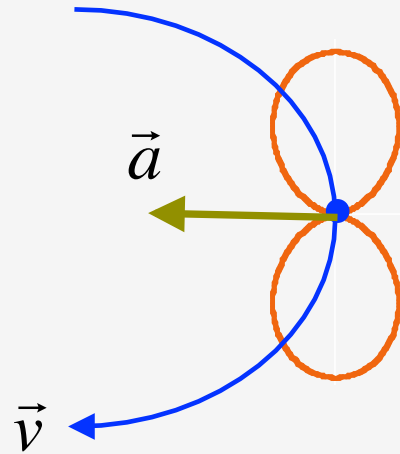
Polarisation



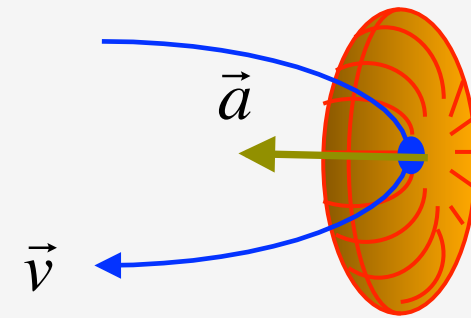
Time structure

S.R. angular distribution (a)

$$v \ll c$$



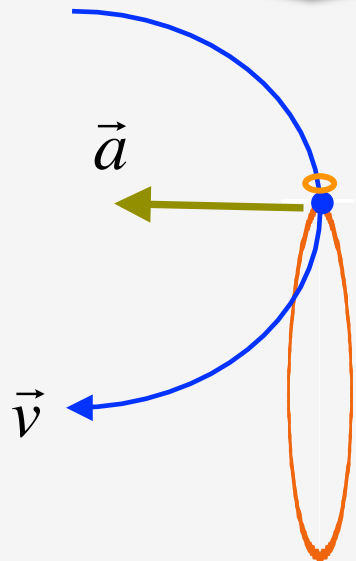
Classical dipole emission pattern



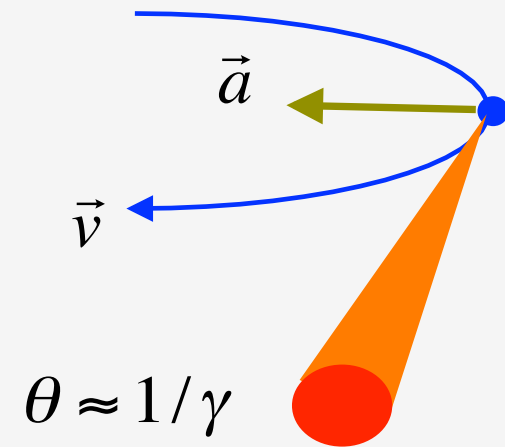
Top view

Perspective

$$v \approx c$$



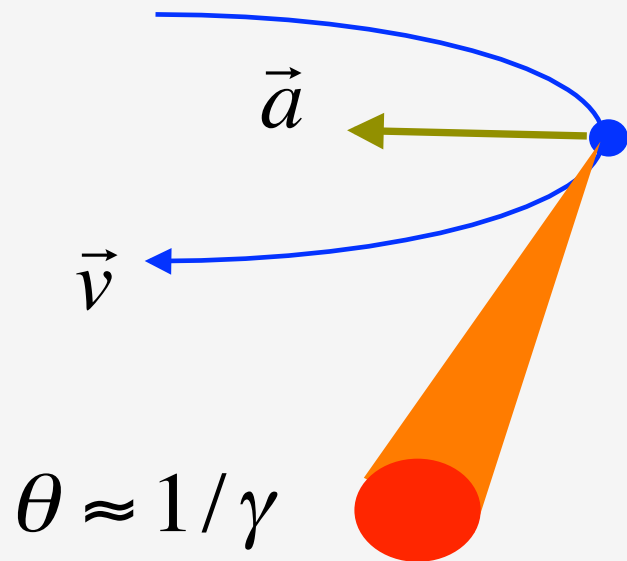
Relativistic emission pattern



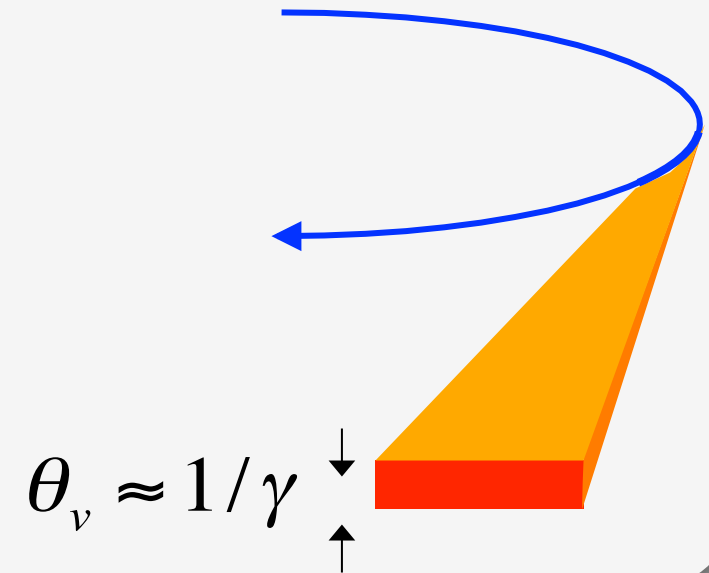
$$\theta \approx 1/\gamma$$

S.R. angular distribution (b)

Instantaneous emission
from one electron



Electron beam
in bending magnet



ESRF:

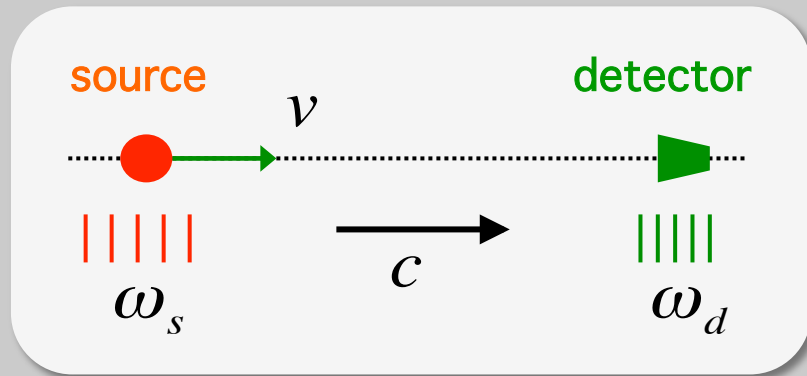
$$W = 6 \text{ GeV}$$

$$\gamma = \frac{W}{m_0 c^2} \approx 12000$$



$$\theta \approx \frac{1}{\gamma} \approx 10^{-4} \text{ rad} \approx 0.005^\circ$$

Relativistic Doppler effect



v = approaching velocity
(detector .vs. source)

c = electrom. wave velocity
(indep. of reference)

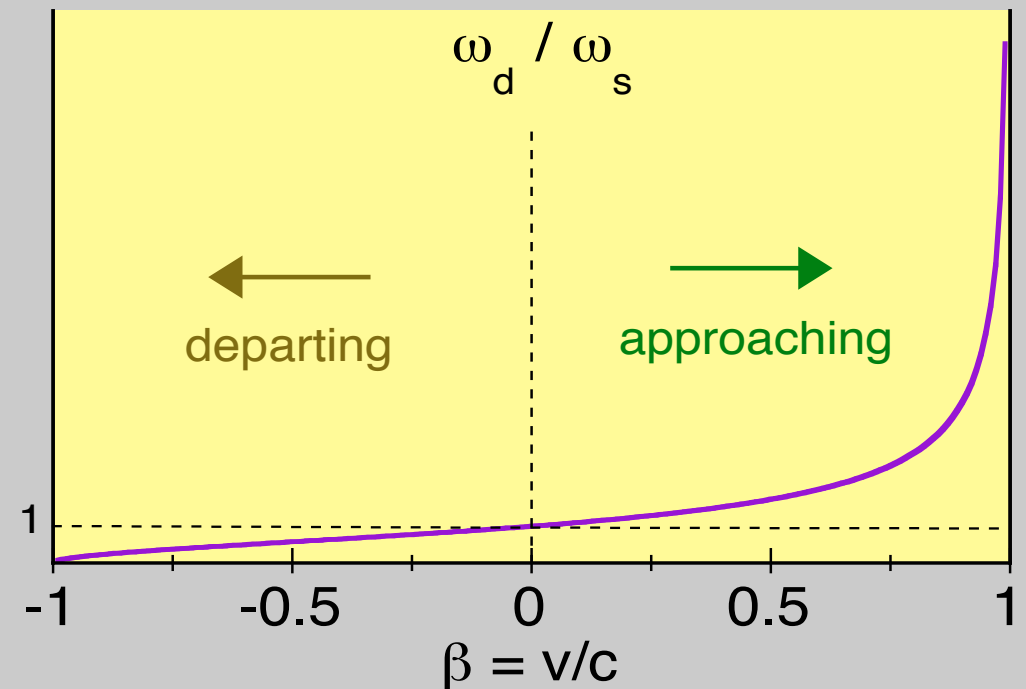
Lorentz-invariance

$$kx - \omega t = k' x' - \omega' t'$$

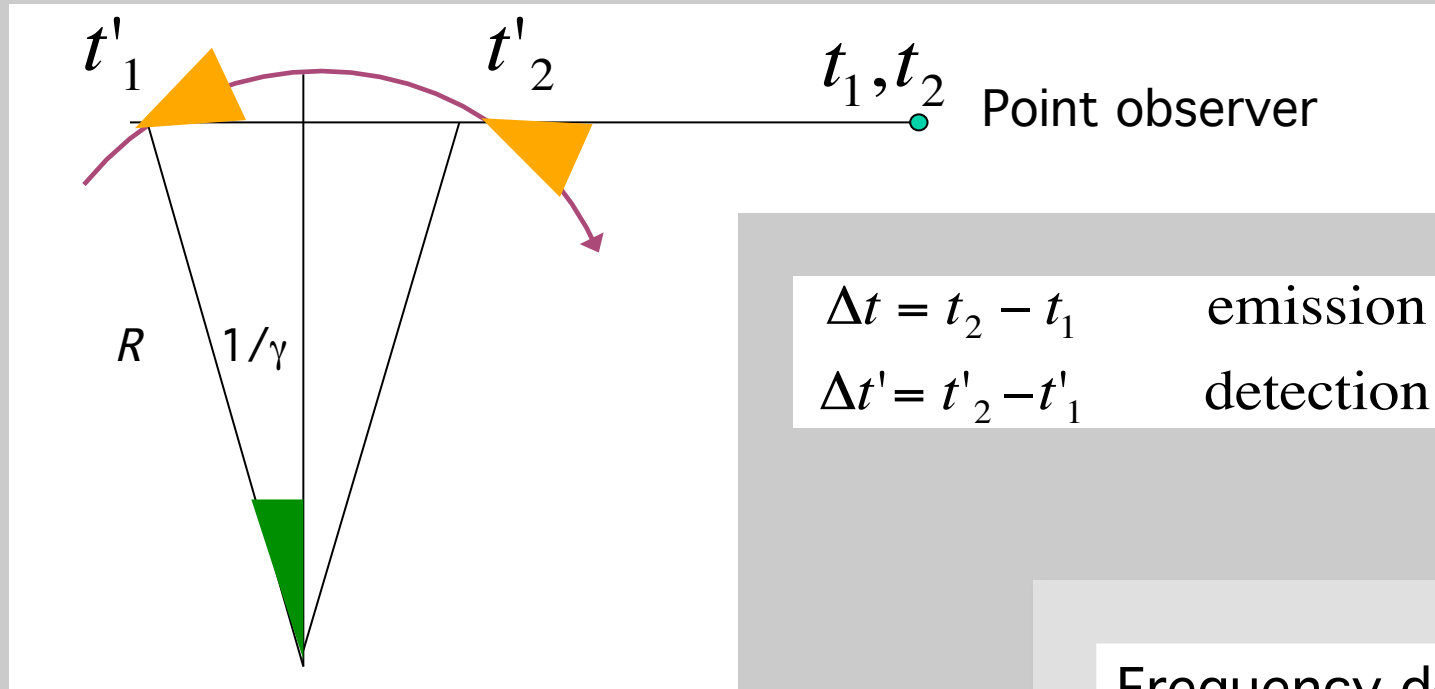


Frequency (energy) shift

$$\omega_d = \omega_s \sqrt{\frac{1 + v/c}{1 - v/c}} = \omega_s \gamma (1 + v/c)$$



S.R. spectral properties (a)

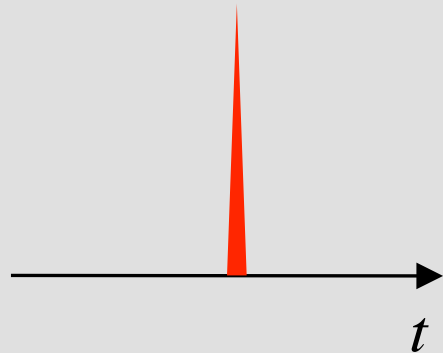


$\Delta t = t_2 - t_1$ emission time
 $\Delta t' = t'_2 - t'_1$ detection time

Time domain

Short time pulse

$$\Delta t \cong \frac{4}{3} \frac{R}{c\gamma^3}$$

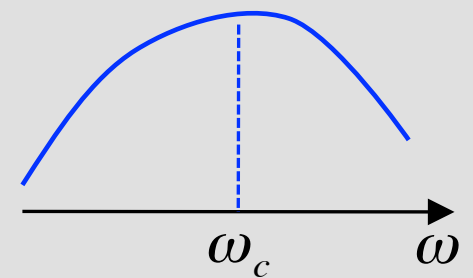


Frequency domain

High frequencies

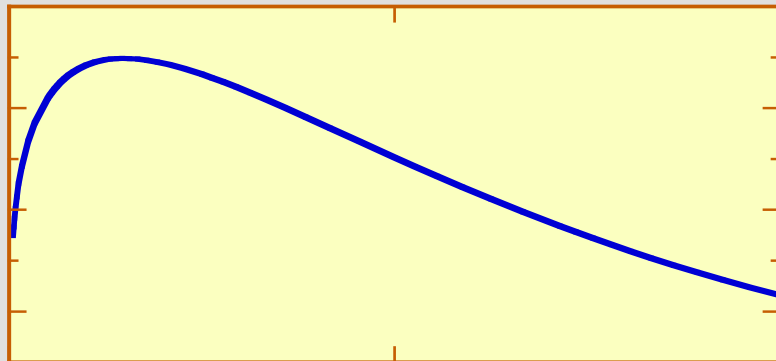
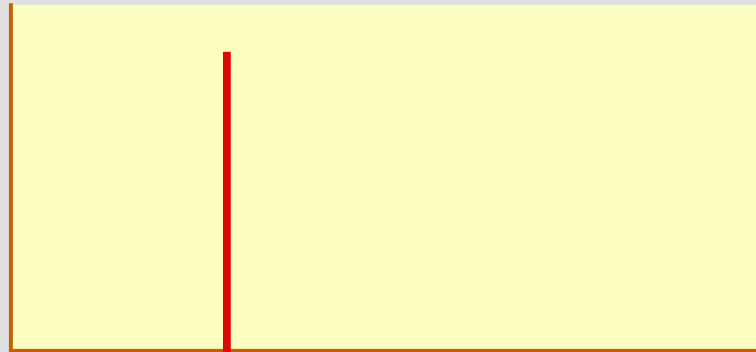
$$\omega_c \approx \frac{3c\gamma^3}{2R}$$

Broad spectrum



S.R. spectral properties (b)

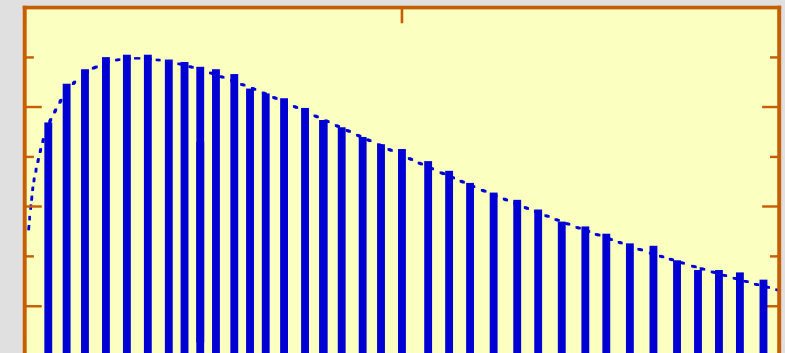
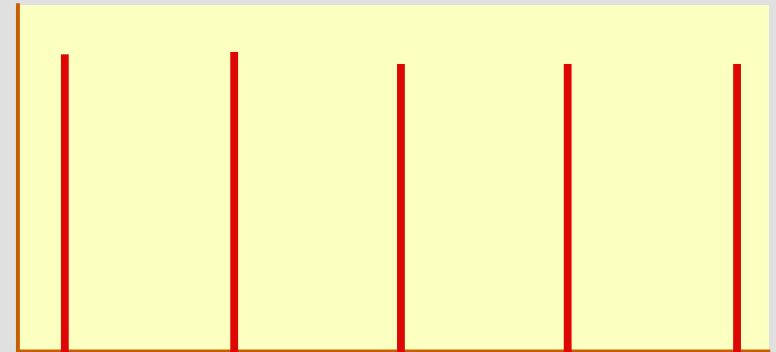
Single emission
from circular arc



ω_c

Time

Periodic emission
from a circular
trajectory



ω_{rev}

ω_c

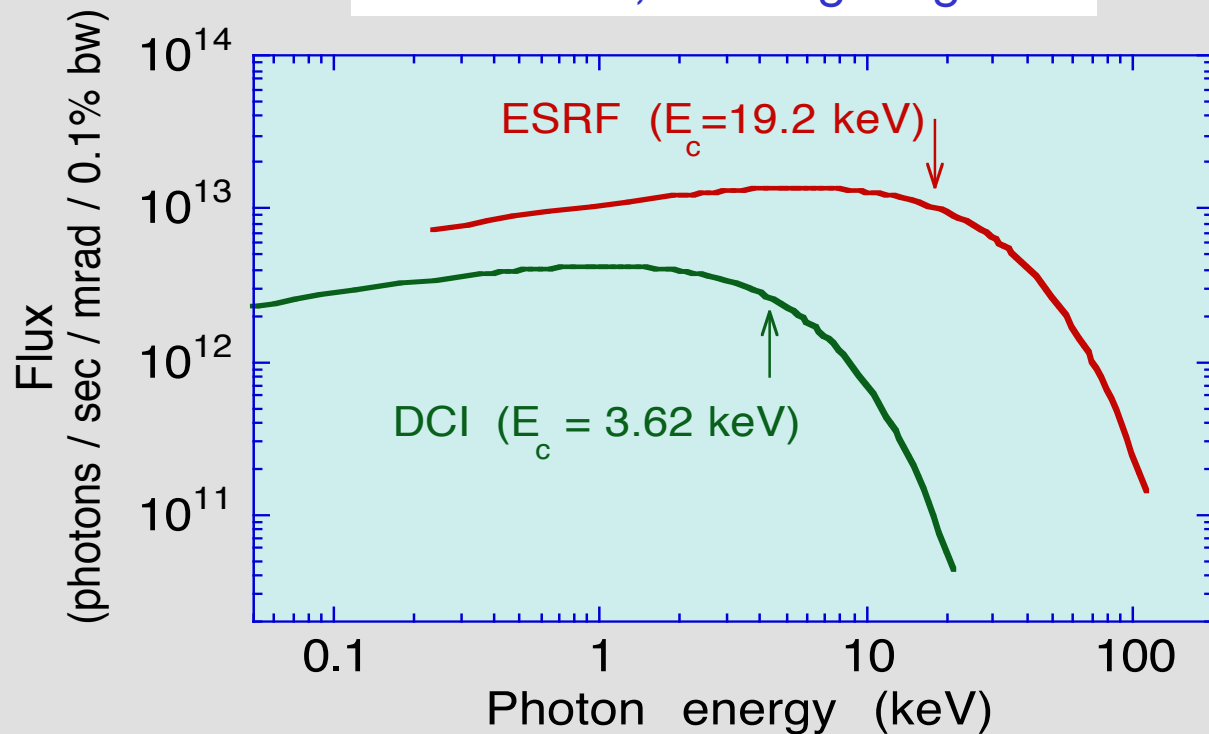
Frequency

S.R. emission spectra

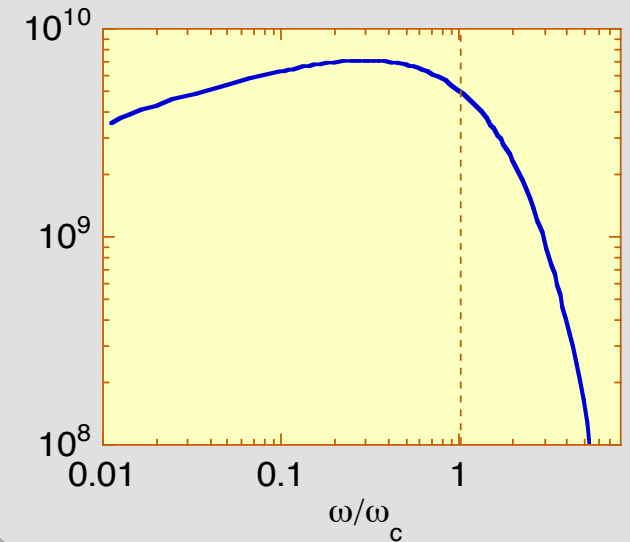
Photons/s/mrad/ 0.1% $\Delta\lambda/\lambda$

$$\text{Flux} = I \gamma F_1$$

$I=100$ mA, bending magnets



$F_1 =$ S.R. universal curve



S.R. polarization

σ - Horizontal component (in the orbit plane)

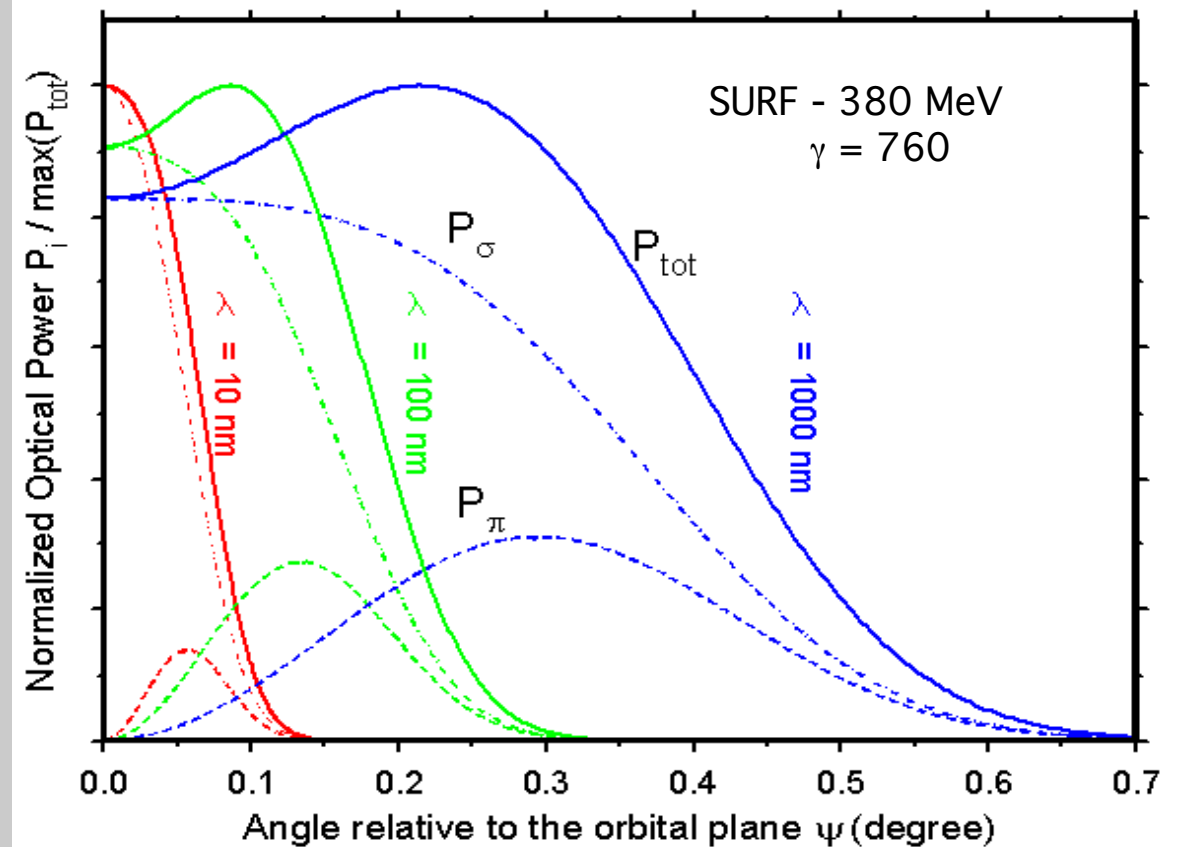
π - The vertical component:
increases with angle
decreases with photon energy

$\pm \frac{\pi}{2}$ dephasing

↓
Elliptical polarisation

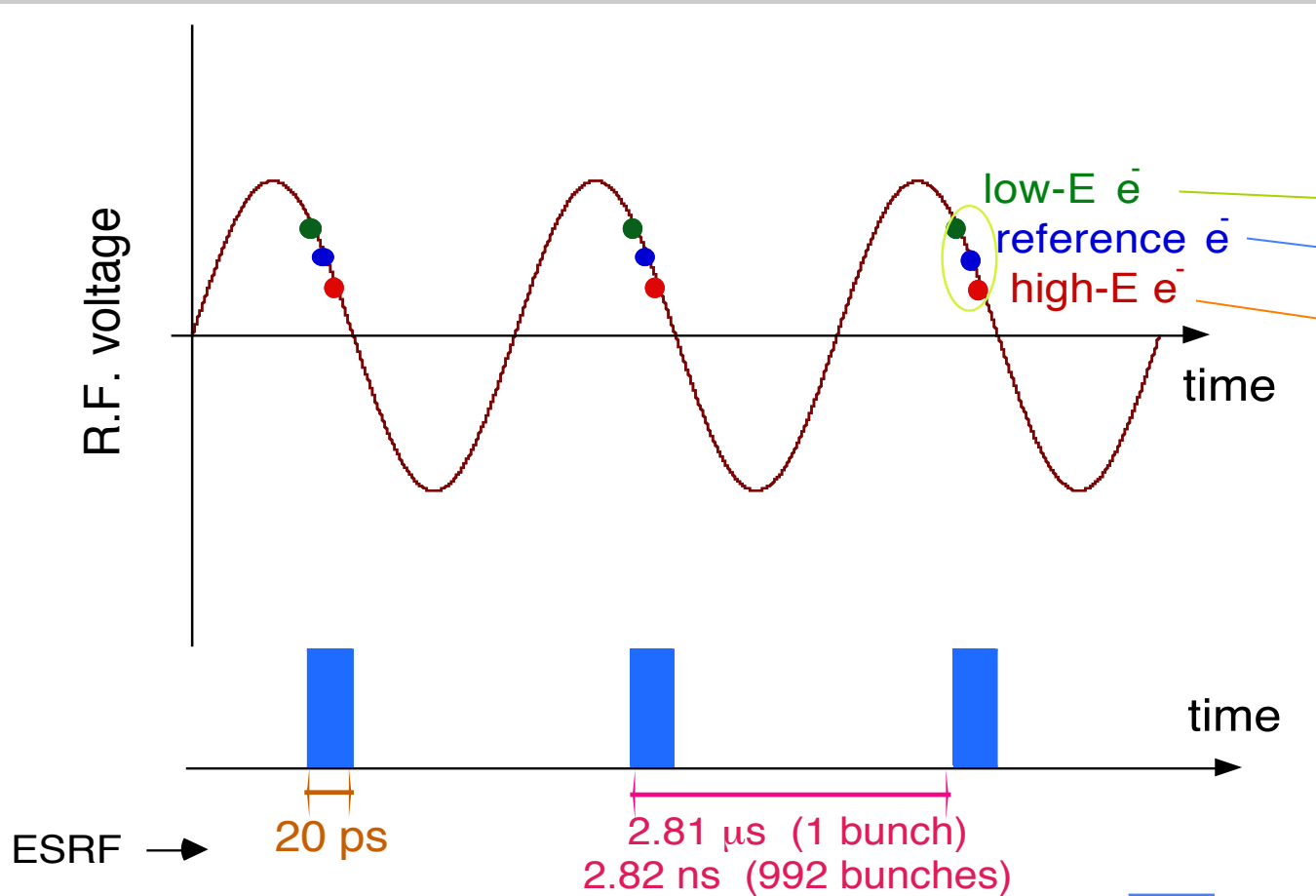
Vertical divergence


$$\psi \approx \frac{1}{\gamma} \quad \text{for} \quad E = E_c$$

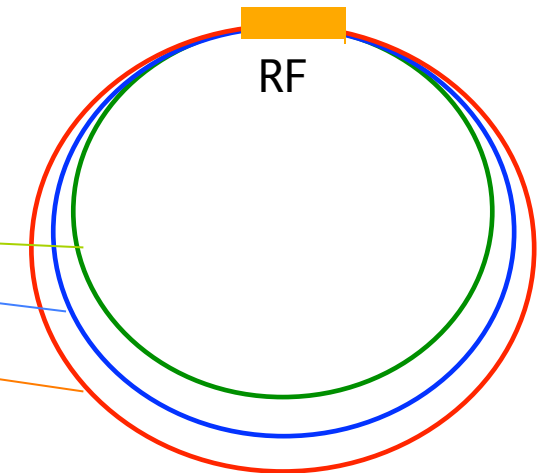


S.R. time structure

Phase-focussing in RF cavities



Orbits



$$v \approx c; \quad \rho = \frac{m_0 c}{eB} \gamma$$

Bunched structure:

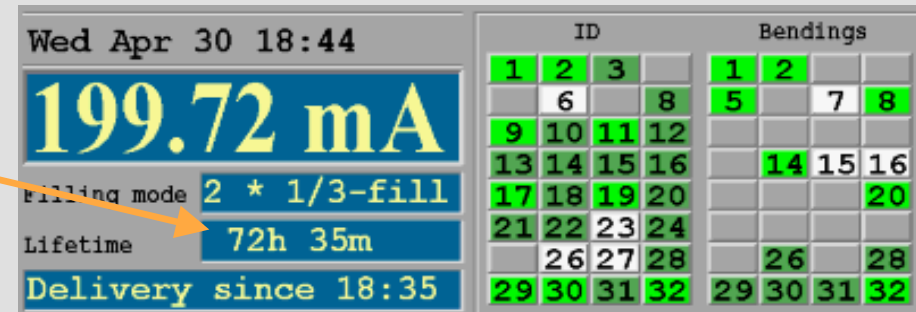
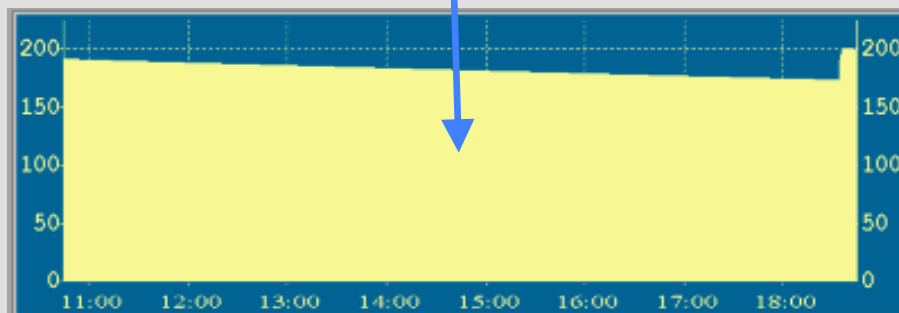
- of electron beam
- of S.R. emission

e^\pm beam lifetime

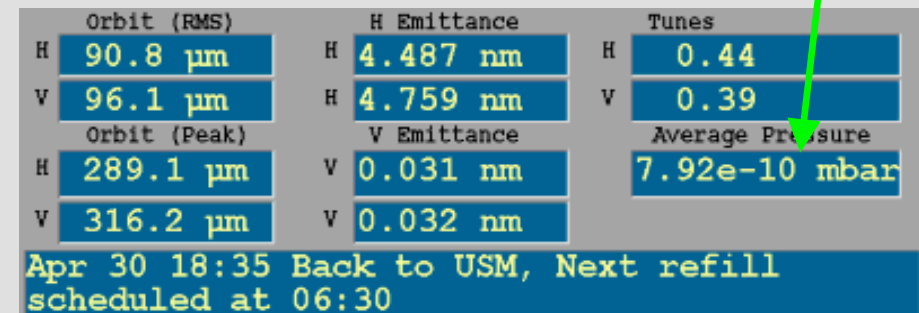
- Collisions with residual gas (photon-stimulated desorption)
- Occasional large energy losses through S.R. emission
- Non-linear resonances (anharmonic betatron oscillations)
- Toushek effect (e-e scattering inside each bunch)

$$\tau = \frac{I(t)}{dI/dt}$$

$$I(t) = I_0 \exp[-t/\tau]$$

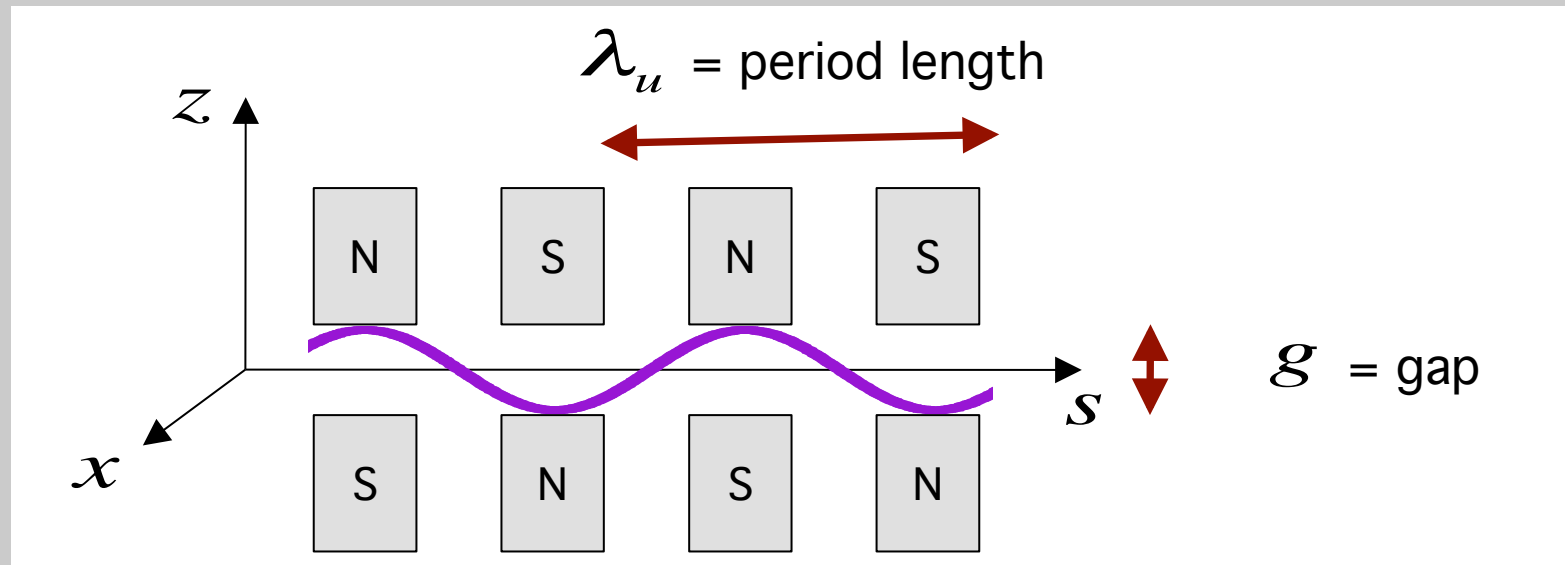


ESRF



S. R. from insertion devices

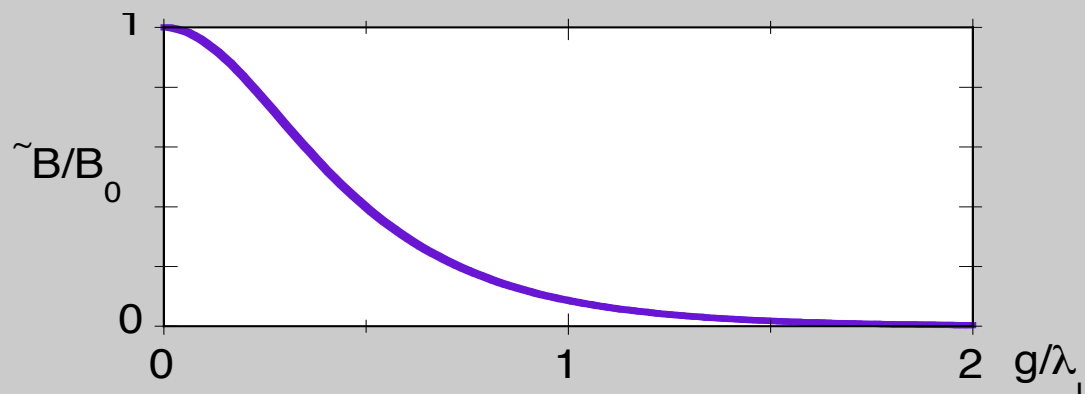
Alternating magnetic fields



Vertical magnetic field

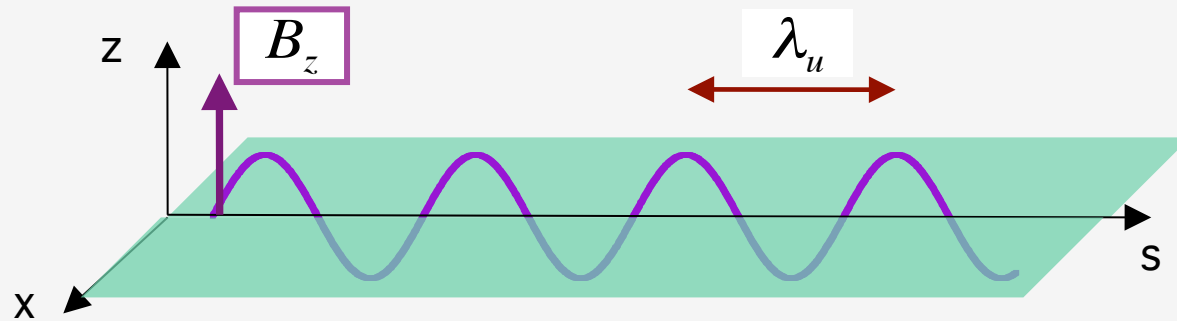
$$B_z(s) = \frac{B_0}{\cosh(\pi g / \lambda_u)} \cos(k_u s) = \tilde{B} \cos(k_u s)$$

Gap-period
relation



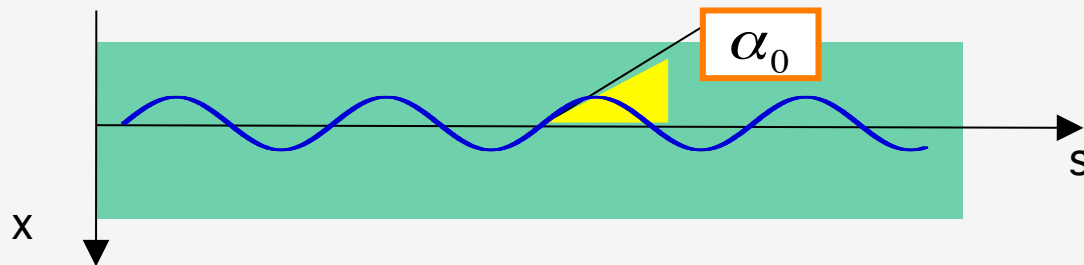
$$k_u = \frac{2\pi}{\lambda_u}$$

Magnetic field effects



Oscillating
vertical magnetic field

$$\vec{F} = e\vec{v} \times \vec{B}$$



Transverse beam oscillation

$$\alpha_0 = K \frac{1}{\gamma}$$

Wiggler/undulator
parameter

$$K = \frac{eB\lambda_u}{2\pi c m_e}$$

The K parameter

W/U properties

SR divergence

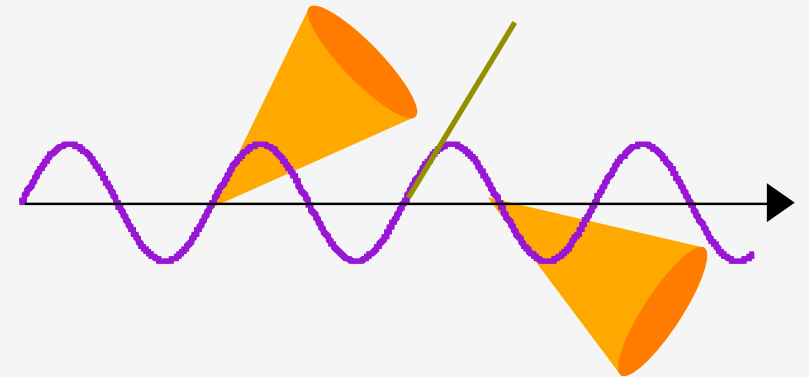
$$\alpha_0 = K \frac{1}{\gamma}$$

$$K = \frac{eB\lambda_u}{2\pi cm_e} = 0.934 \times B[\text{T}] \times \lambda_u[\text{cm}]$$

$$K > 1, \quad \alpha_0 > \frac{1}{\gamma}$$

High-K devices

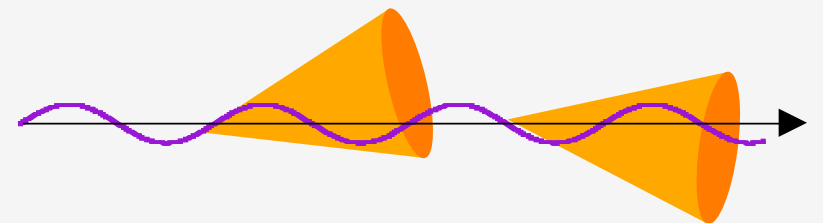
WIGGLERS



$$K \leq 1, \quad \alpha_0 \leq \frac{1}{\gamma}$$

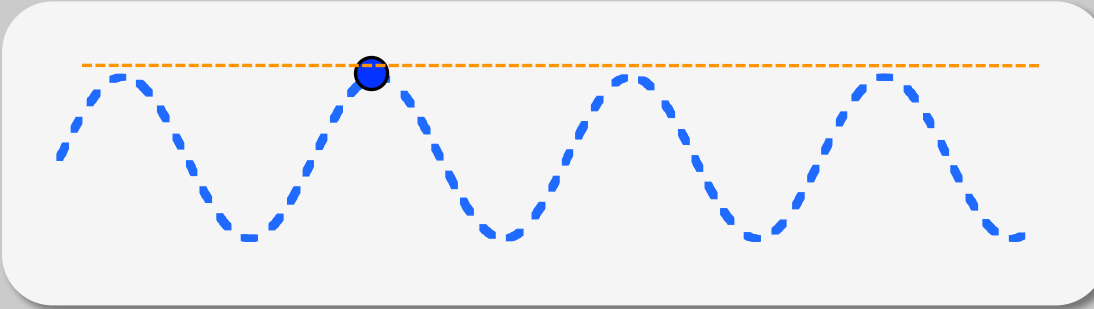
Low-K devices

UNDULATORS

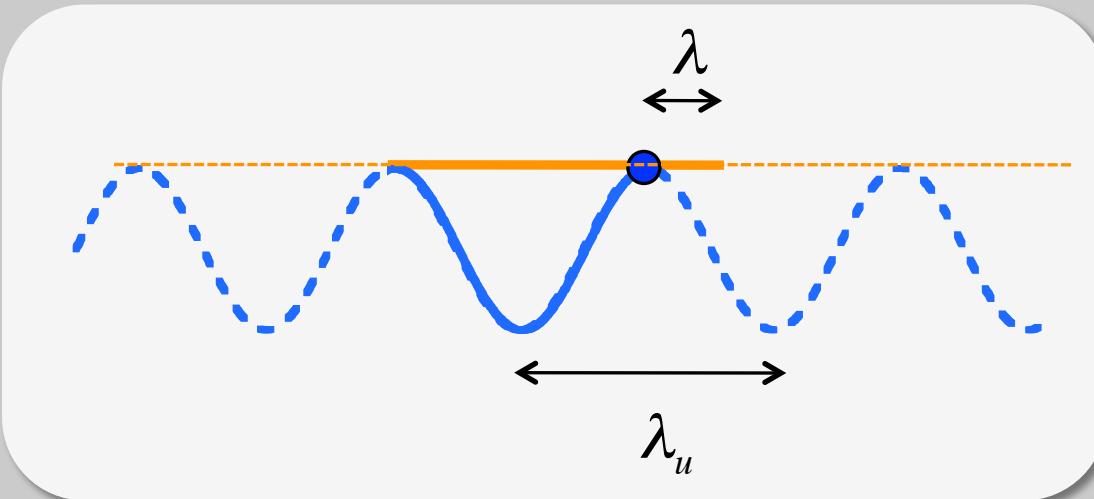


Undulator: interference effect

time t_1



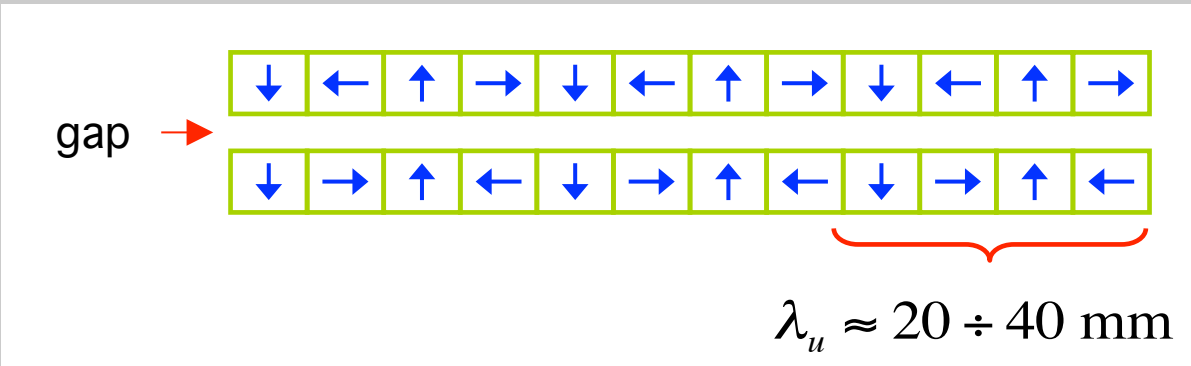
time t_2



Constructive interference for

$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

Properties of undulator radiation (a)



$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

tunability:
gap \Rightarrow B \Rightarrow K

Angular
red-shift

- Large N
- $K < 1$



Interference
(monochromatic radiation)

Properties of undulator radiation (b)

- Interference → Flux $\propto N^2$ (in forward direction)

N = number of wiggles

- Electron motion not perfectly sinusoidal → higher order harmonics

$$\omega_1 \approx \frac{4\pi c \gamma^2 / \lambda_u}{1 + K^2 / 2 + \gamma^2 \theta^2}$$

$$\omega_n = n\omega_1$$

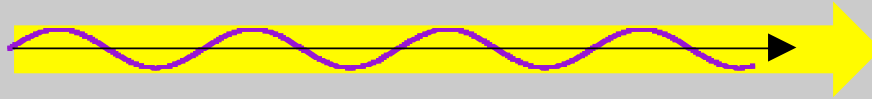
- Finite wave-train of radiation emitted by each electron → bandwidth

$$\frac{\Delta\omega_n}{\omega_n} \approx \frac{\gamma^2 \delta(\theta^2)}{1 + K^2 / 2}$$

$$\frac{\Delta\omega_n}{\omega_n} \approx \frac{1}{nN}$$

Low-K .vs. high-K devices

Low-K



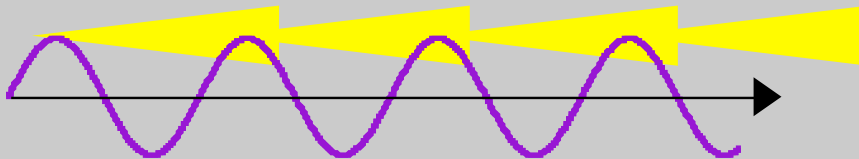
Coherent superposition
Interference

$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

Increasing K:

- λ increases
- relevance of harmonics
- interference effects reduced
- broader spectrum

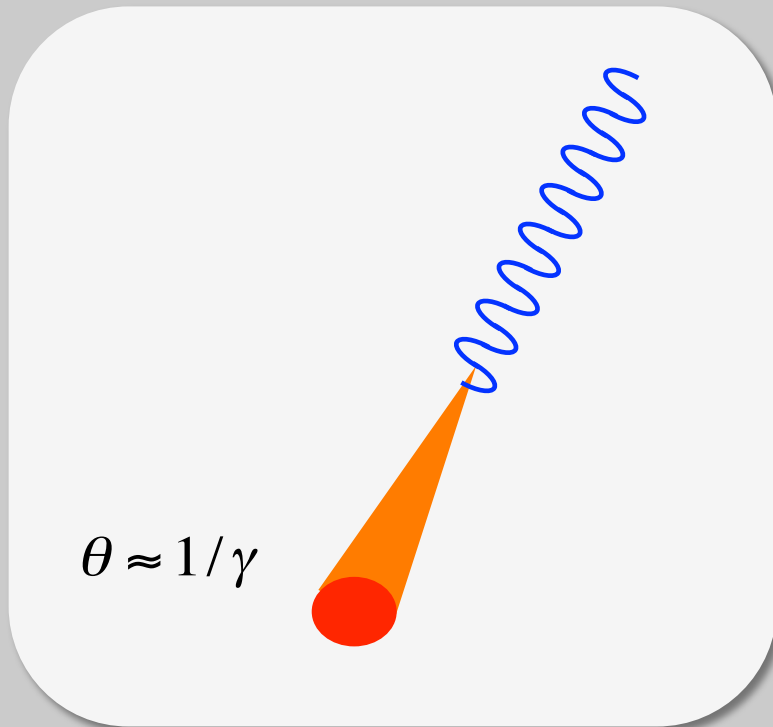
High-K



Incoherent superposition
Continuous spectrum

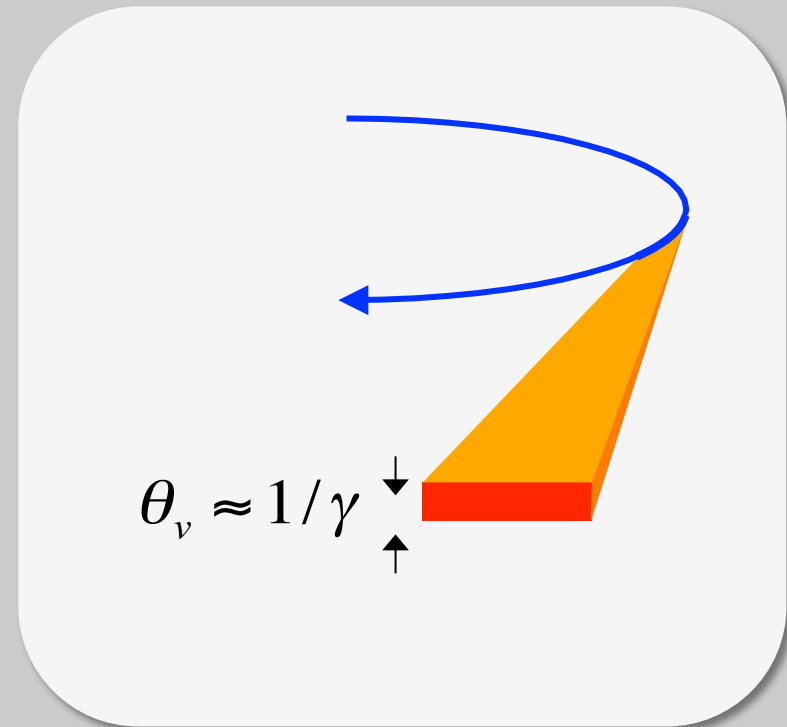
Undulators .vs. bending magnets (a)

Undulator



Vertical and horizontal collimation

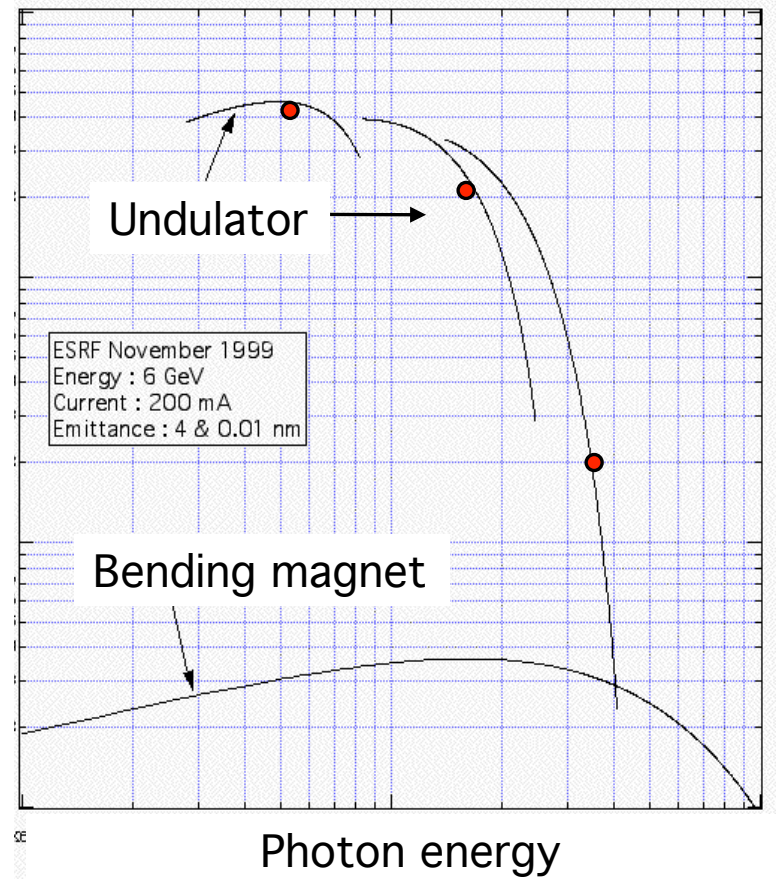
Bending magnet, wiggler



Vertical collimation

Undulators .vs. bending magnets (b)

Flux



Undulator

Strong emission at discrete energies (fundamental + harmonics)

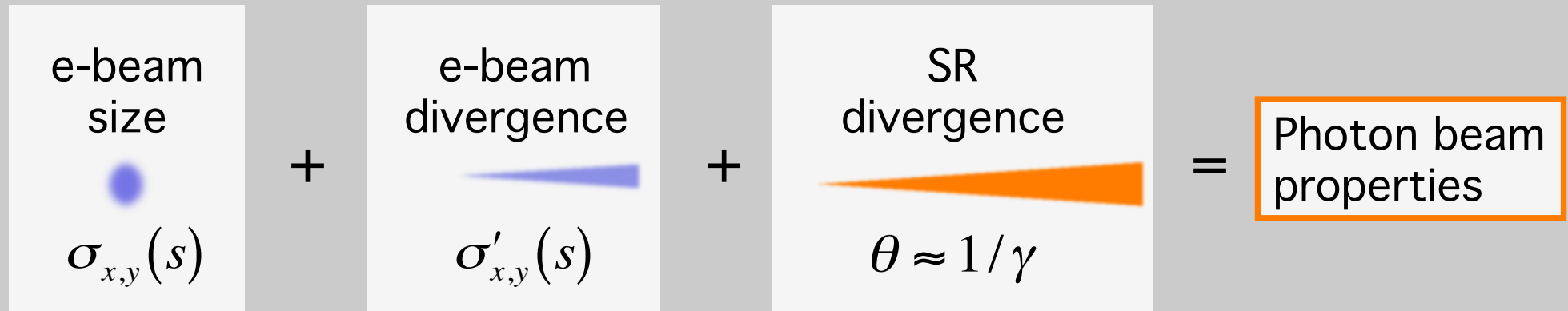
Tunability by varying the gap

Bending magnet, wiggler

Emission over a continuous spectrum

Brilliance of S.R.sources

Photon beam properties



ESRF	Horizontal		Vertical	
	σ_x [μm]	σ'_x [μrad]	σ_y [μm]	σ'_y [μrad]
Even I.D.	415	51	8.6	2.9
Odd I.D.	10.3	108	8.6	2.9

S.R. divergence $\theta \approx 1/\gamma \approx 100 \mu\text{rad}$

Photon beam parameters

Brilliance

$$\frac{\text{photons}}{\text{s mm}^2 \text{ mrad}^2 \text{ bandwidth}}$$

↑ ↑ ↑

source size solid angle 0.1% $\Delta\lambda/\lambda$

Integrating over source size

+

Integrating over vert. angle

+

Integrating over horiz. angle

+

Integrating over bandwidth

Brightness

$$\frac{\text{photons}}{\text{s} \cdot \text{mrad}^2 \cdot \text{bandwidth}}$$

Spectral flux

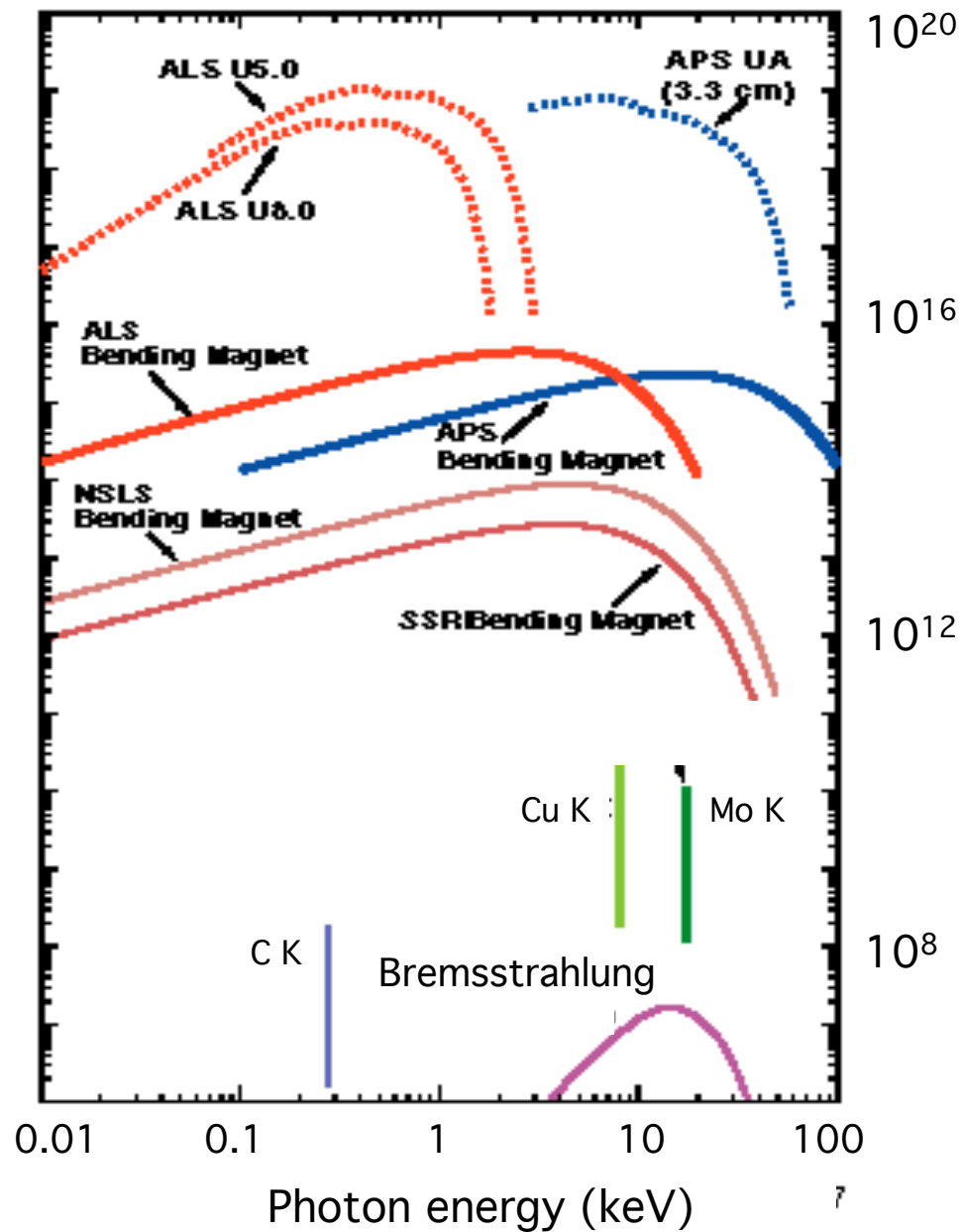
$$\frac{\text{photons}}{\text{s} \cdot \text{mrad} \cdot \text{bandwidth}}$$

Total flux

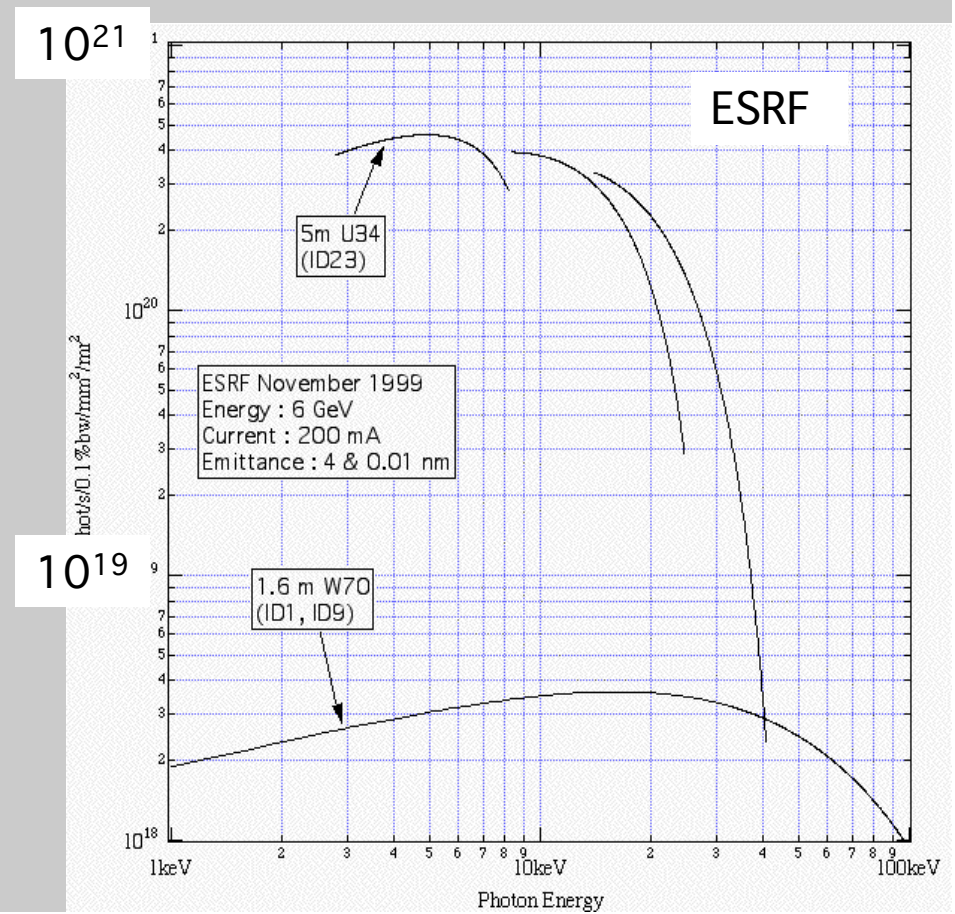
$$\frac{\text{photons}}{\text{s} \cdot \text{bandwidth}}$$

$$\frac{\text{photons}}{\text{s}}$$

Brilliance: comparisons

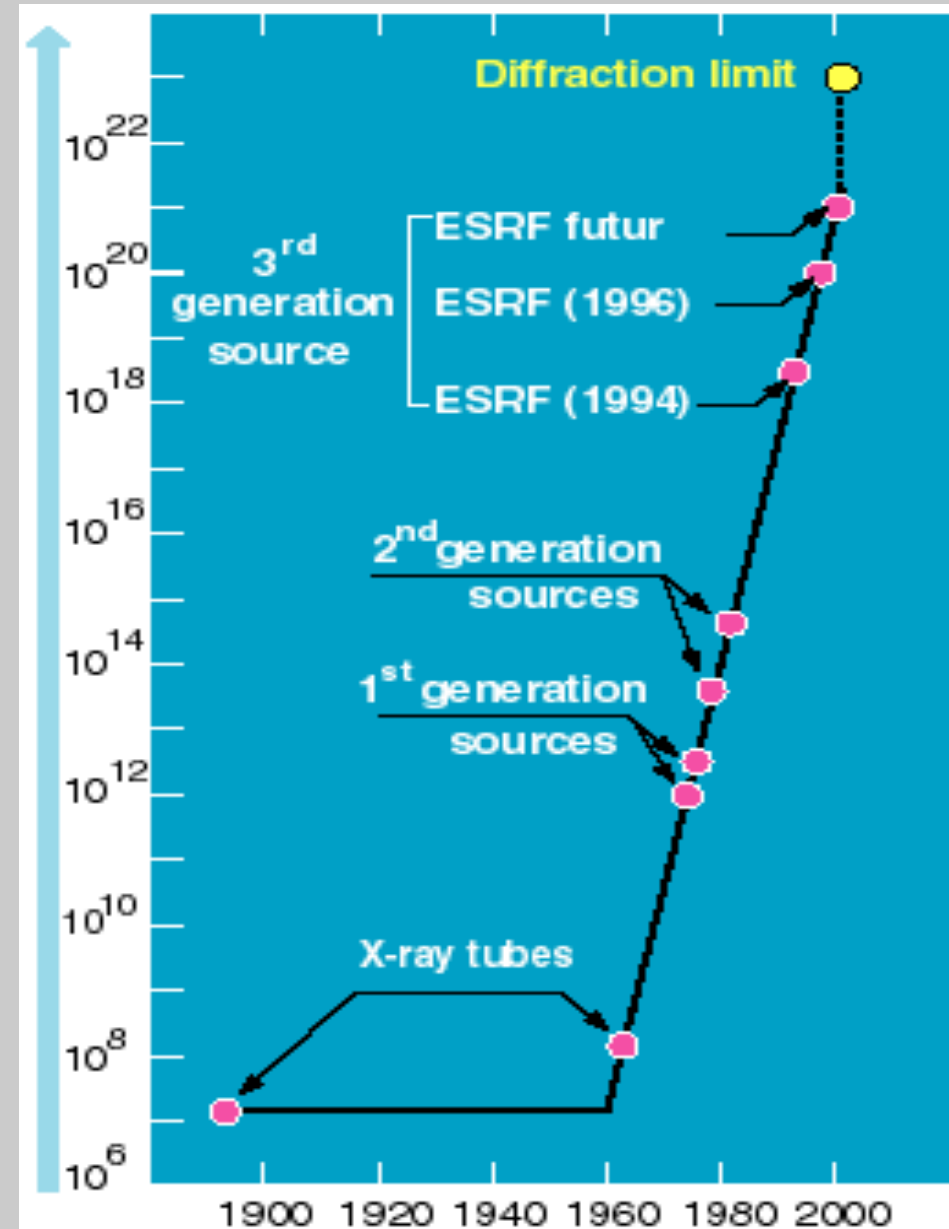


photons
s mm² mrad² bandwidth



Brilliance: time evolution

$$\frac{\text{photons}}{\text{s mm}^2 \text{ mrad}^2 \text{ bandwidth}}$$



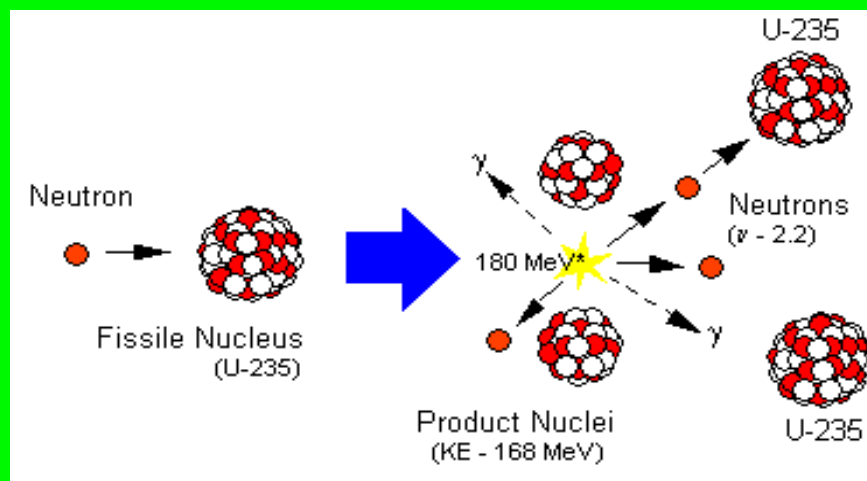
Neutron sources

Neutron production

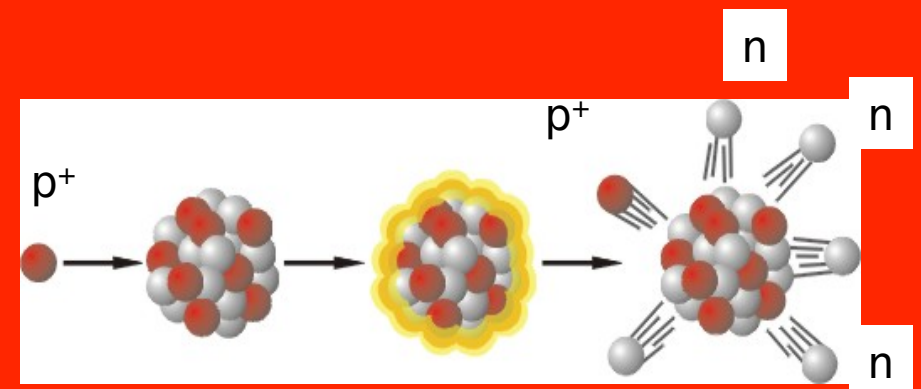
- Radioactive sources
- Neutrons from nuclear reactors
- Neutrons from pulsed accelerators
 - photofission
 - spallation

Most effective for solid state research:

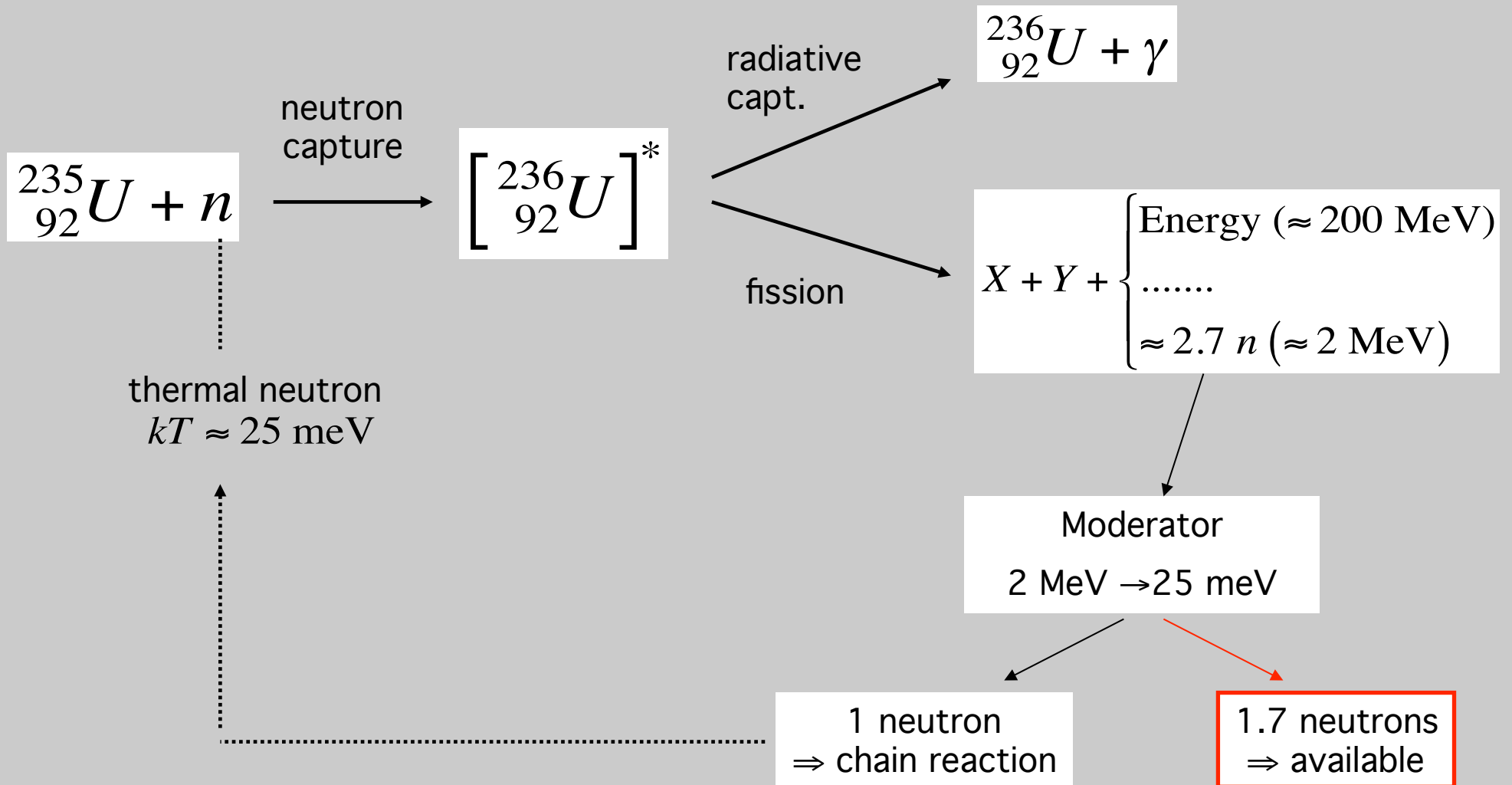
Fission (reactors)



Spallation (accelerators)



Fission

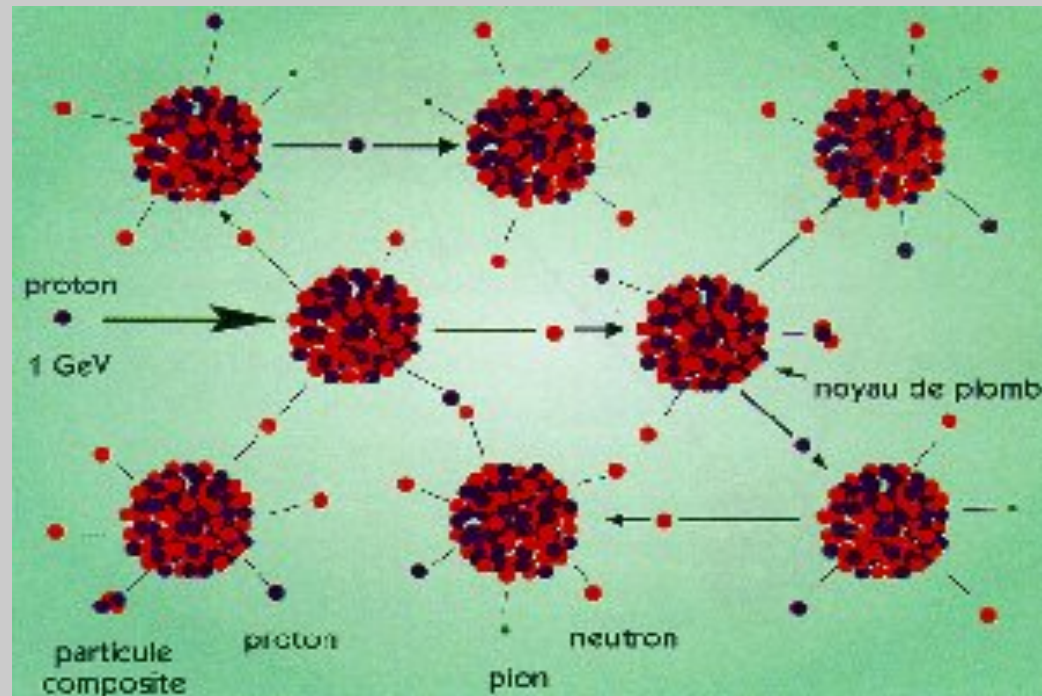


Spallation

Proton accelerator

≈ 1 GeV proton

interaction
with nucleons

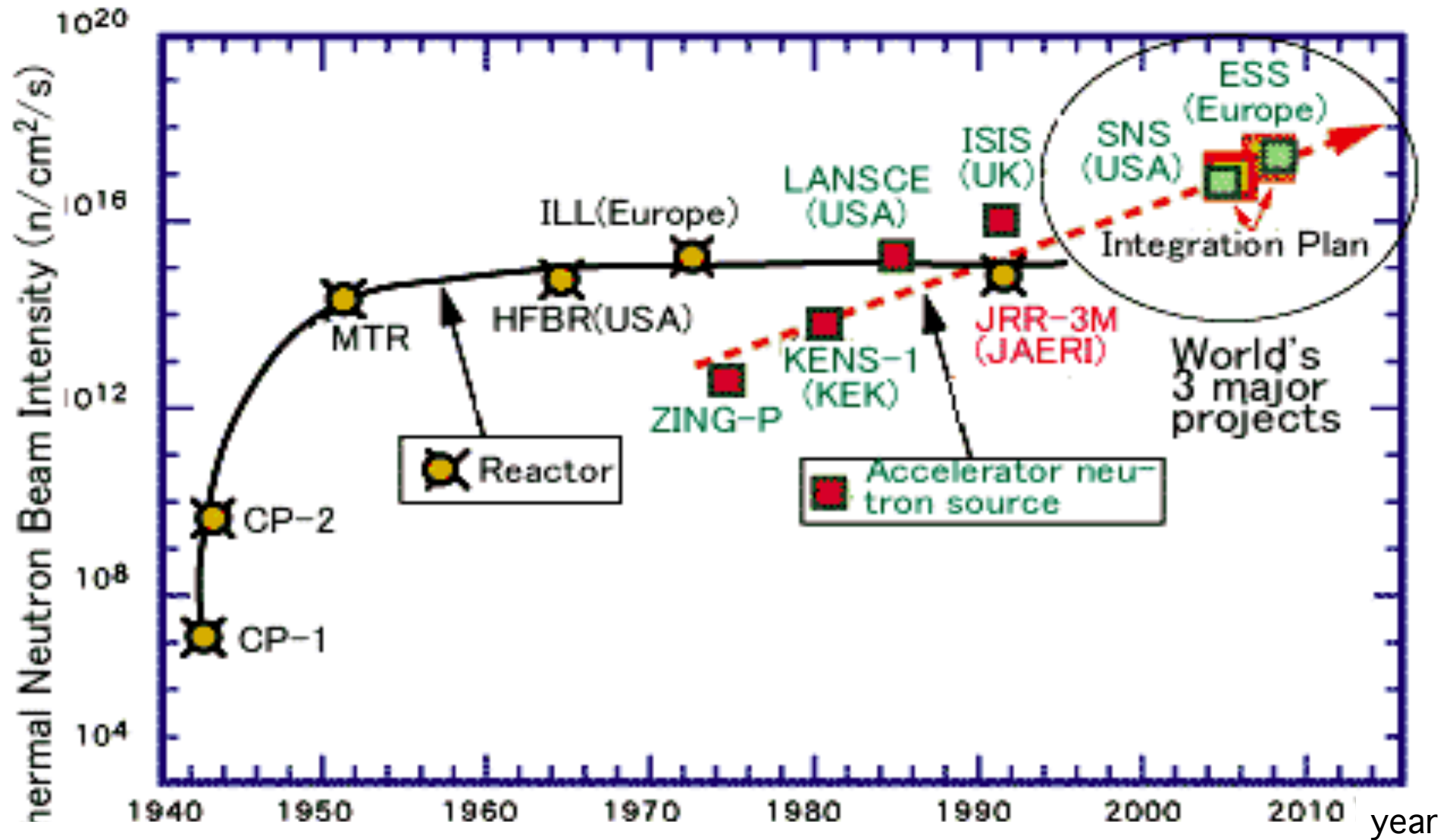


- high-energy particles
- chain-reactions
- excitations of nuclei
- de-excitation (evapor.)

20-40 neutrons
(≈55 MeV)

- Concentrated source ⇒ high flux
- Pulsed source

Neutron sources

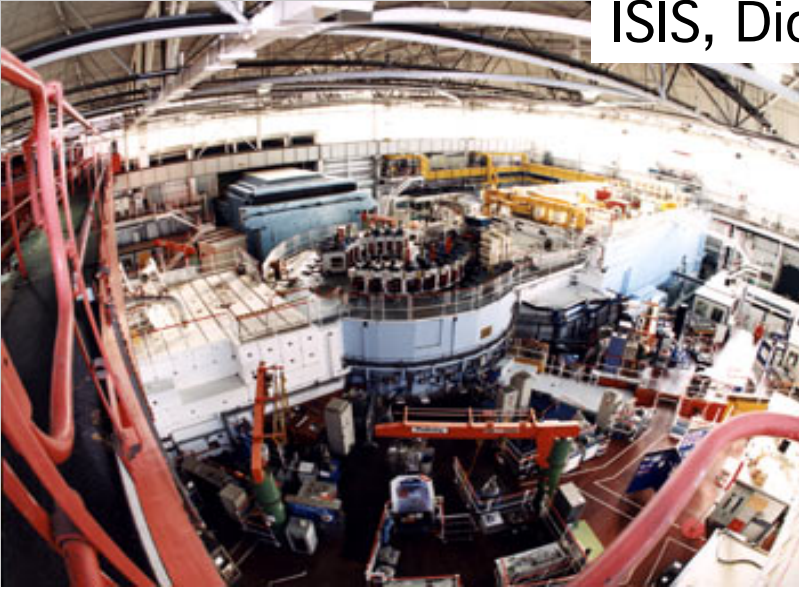
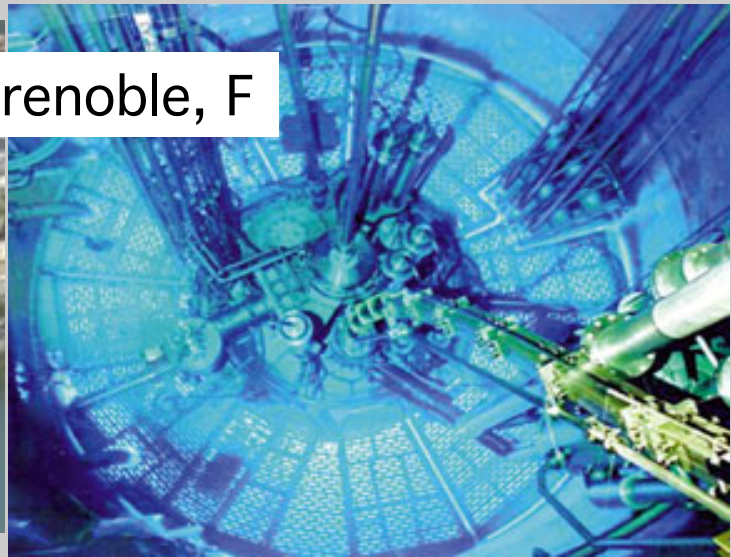


Largest European Neutron Labs



ESRF(SR)

ILL Grenoble, F



ISIS, Didcot, UK

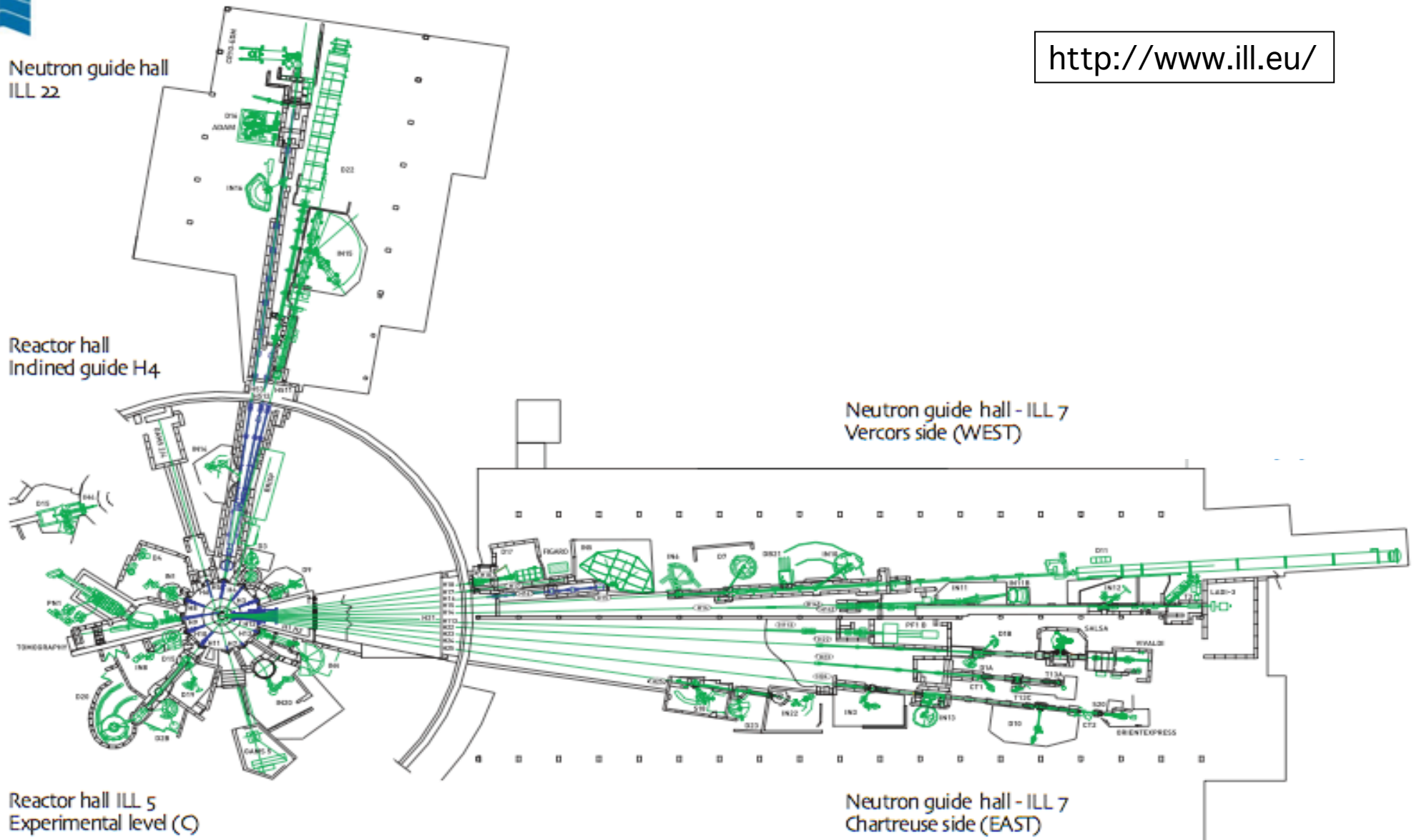


Diamond (SR)

ILL reactor source



<http://www.ill.eu/>



Neutron guide hall ILL 22

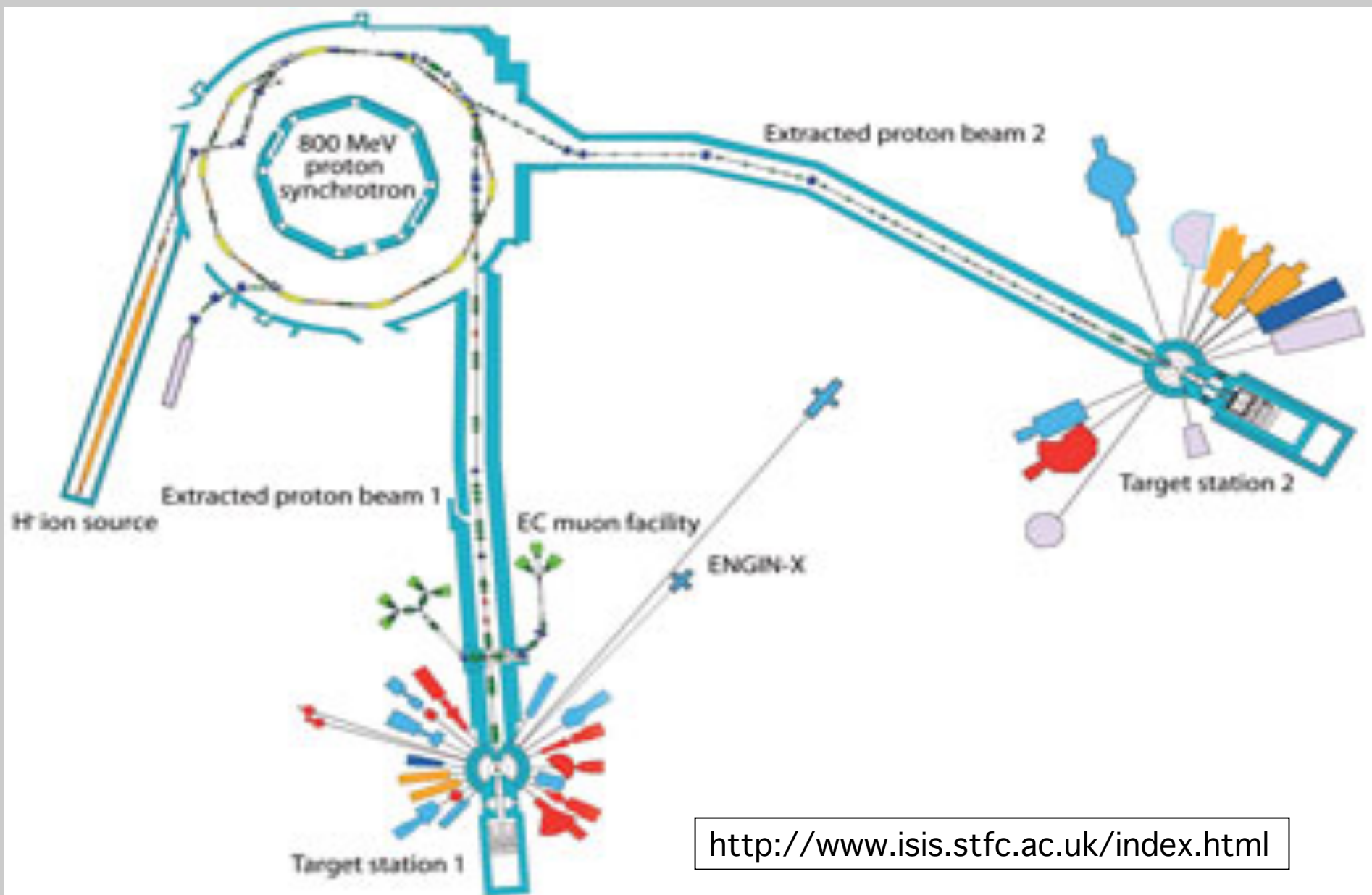
Reactor hall Indined guide H4

Neutron guide hall - ILL 7 Vercors side (WEST)

Reactor hall ILL 5 Experimental level (C)

Neutron guide hall - ILL 7 Chartreuse side (EAST)

ISIS spallation source



<http://www.isis.stfc.ac.uk/index.html>

Energy of neutrons

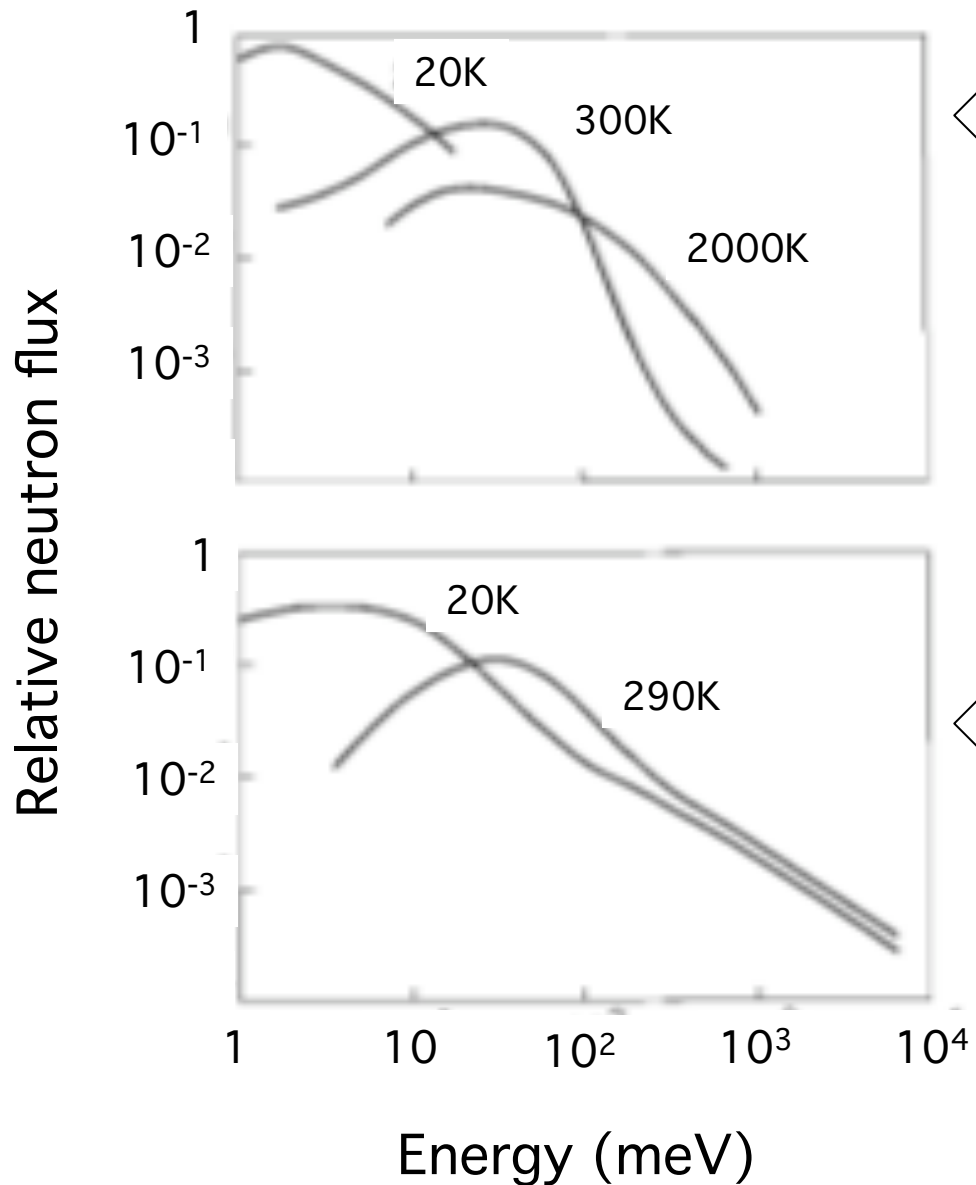
Moderation:

Neutrons are slowed down in moderators, where they are brought to thermal equilibrium through inelastic collisions with light atoms (H, D, Be)

The Table refers to the peak values of the Maxwell equilibrium distribution.

	<u>Ultra-cold</u>	<u>Cold</u>	<u>Thermal</u>	<u>Hot</u>	<u>Epi-thermal</u>
Energy (meV)	0.00025	1	25	250	1000
Temperature (K)	0.003	12	290	2900	12000
Wavelength (Å)	570	9	1.8	0.57	0.29
Velocity (m/s)	6.9	440	2200	6900	14000

Energy distribution of neutrons



Reactor sources

CONTINUOUS PRODUCTION
Energy selection
by crystal monochromators
(Bragg law)

Spallation sources

Higher energies available

PULSED PRODUCTION
Energy selection
by time-of-flight techniques

Neutrons and X-rays properties (b)

	Thermal neutrons	X-rays (synchrotron)	X-rays (anodes)
Energy (eV)	10^{-1} eV	10^4 eV	10^4 eV
Wavelength (Å)	0.5 - 10	0.5 - 5	0.5 - 2
Flux (part./cm ² /s)	10^{10} - 10^{15}	10^{20} - 10^{25}	10^{16} - 10^{17}
Sample volume (mm ³)	1 - 10	10^{-5} - 10^{-1}	10^{-3} - 10^{-2}
Tunability	yes	yes	no
Beam divergence	5 mr	10^{-1} mr	5 mr
$\Delta\lambda/\lambda$	10^{-3}	10^{-5} - 10^{-4}	10^{-3}
Absorption	weak	medium	medium